Stress Analysis for Byrd glacier, East Antarctica

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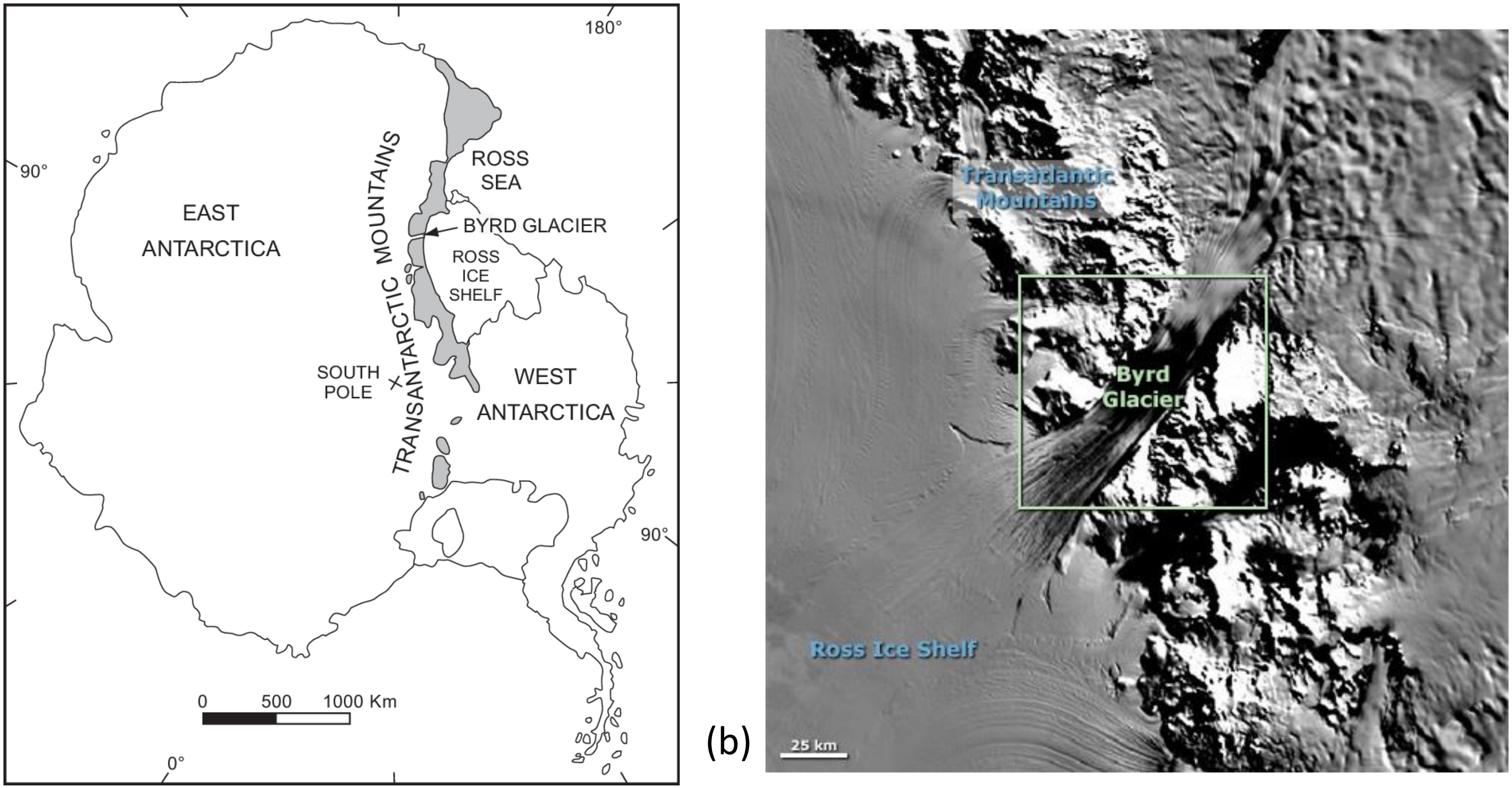
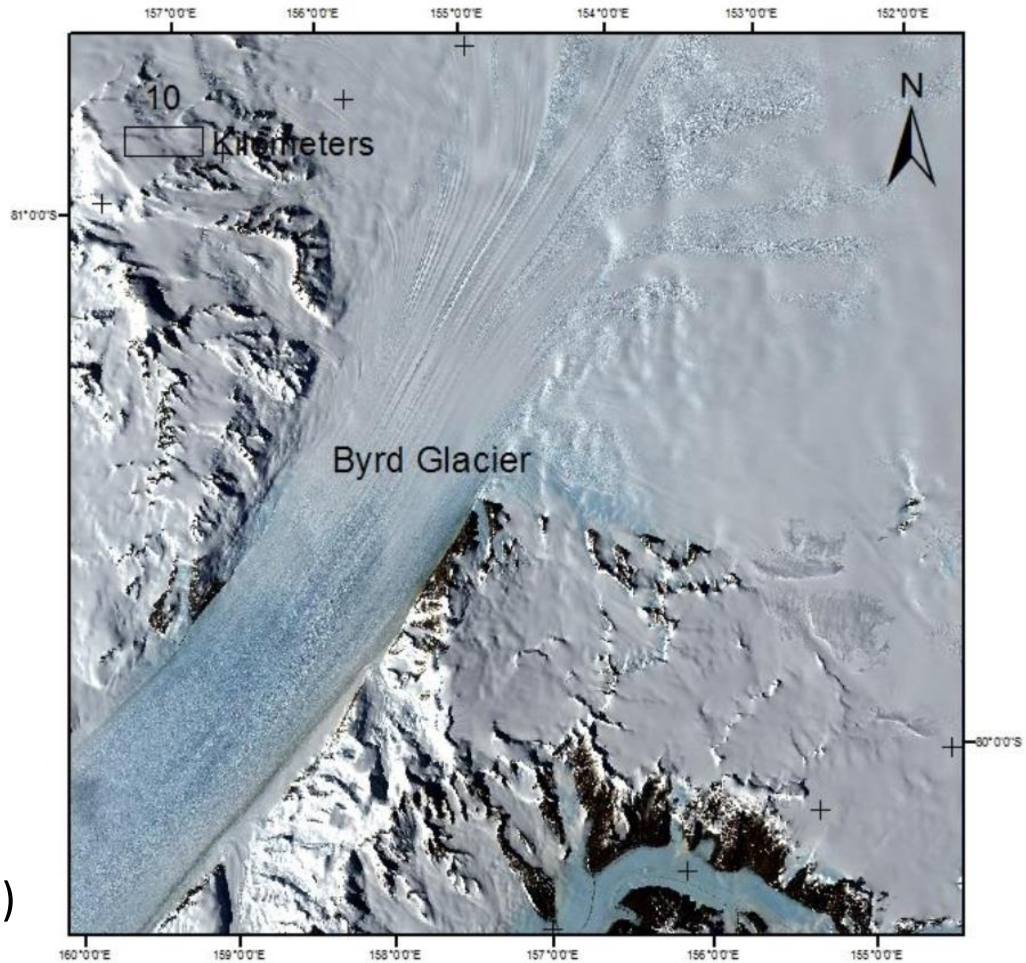
Byrd glacier

- One of the largest outlet glaciers of Antarctica
- Catchment basin with area ~1 Mkm^2 and the ice being funnelled into a ~20 km wide and ~100 km long fjord through the transantarctic mountains and thereafter diverging into the Ross ice Shelf



(a) (b)

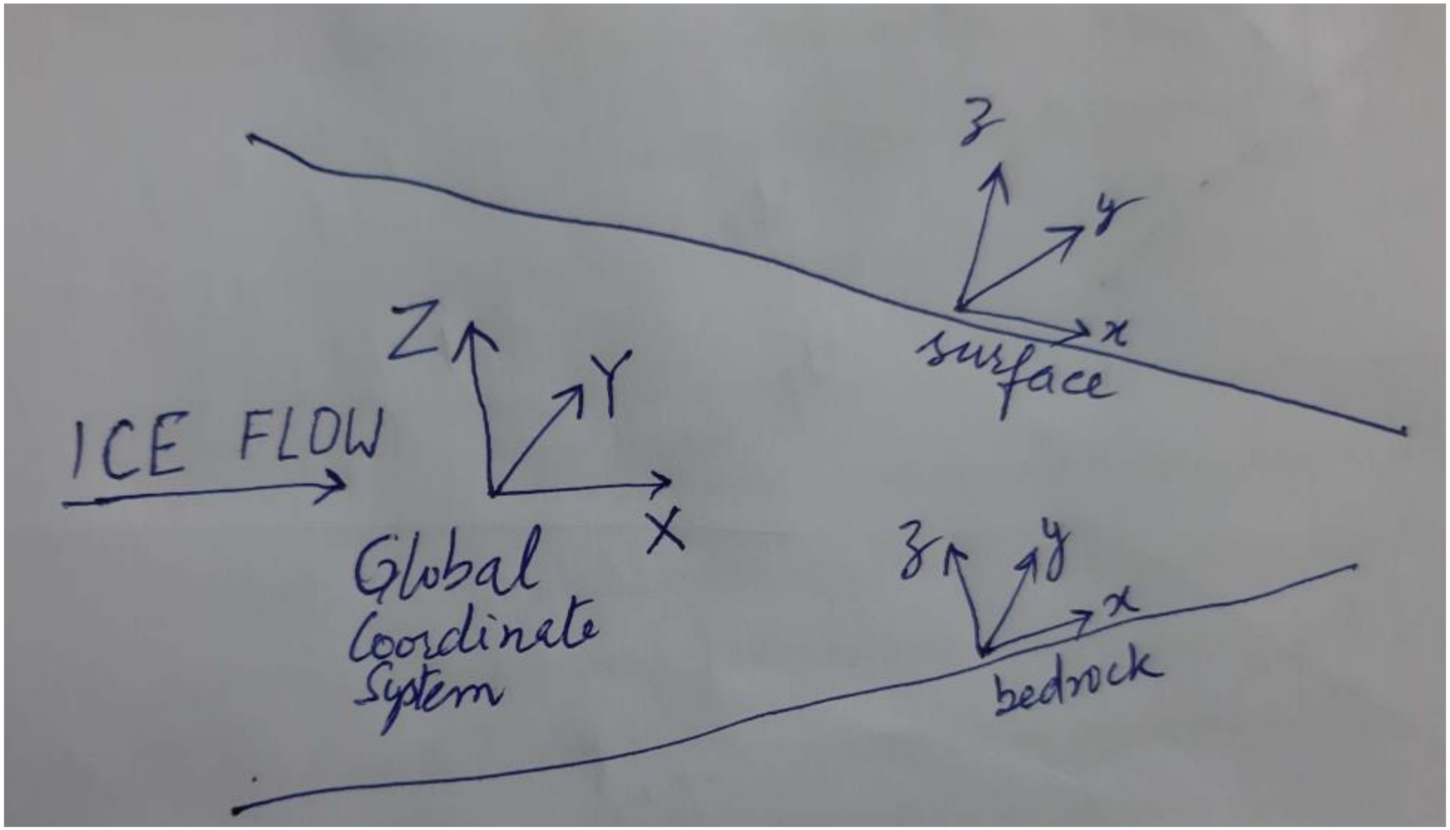
Fig. 2(a, b) Location of Byrd glacier in East Antarctica (Source: https://www.google.co.in/search?q=byrd+glacier&source=lnms&tbm=isch&sa=X&ved=0ahUKEwiQhZuukMrYAhWIu48KHX_8AfUQ_AUICigB&biw=1422&bih=684#imgrc=iUuMqglwv5uyCM)



(d)

Fig. 2 (c) Map showing location of Byrd glacier in East Antarctica. This TIFF image of Byrd glacier is downloaded from https://visibleearth.nasa.gov/view.php?id=7544

Transformation of stresses and stress gradients calculated for surface coordinate system to bedrock coordinate system



1. **Calculation of stresses from given surface velocity field with velocity derivatives calculated by moving least squares interpolation.**

2. Calculation of gravitational driving force, basal drag, lateral drag and longitudinal stress gradient with respect to surface and bedrock coordinate systems.

- Effective strain rate:
  - $\dot{\epsilon}_e{}^2 = \frac{1}{2}\dot{\epsilon}_{ij}\dot{\epsilon}_{ij}$

    $= \frac{1}{2}\left(\dot{\epsilon}^2{}_{xx} + \dot{\epsilon}^2{}_{yy} + \dot{\epsilon}^2{}_{zz}\right) + \dot{\epsilon}^2{}_{xy} + \dot{\epsilon}^2{}_{xz} + \dot{\epsilon}^2{}_{yz}$
  - $\dot{\epsilon}_{ij} = \frac{1}{2}\left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right], \quad i, j = x, y, z$

- Flow law is:
  - $\tau_{ij} = B\dot{\epsilon}_e{}^{\frac{1}{n}-1}\dot{\epsilon}_{ij}$
  - Ice parameters assumed in this study:
    - $\rho = 900\ kg/m^3$, $\mathrm{B} = 2.7\ \mathrm{bar\ a}^{\frac{1}{3}}$

Equation of force equilibrium in x-direction is:

$\sigma_{xx,x} + \sigma_{xy,y} + \sigma_{xz,z} + b_x$=0

where b_x is component of gravitational force per unit volume in x-direction.
Hence,

$\sigma_{xz,z} = -(\sigma_{xx,x} + \sigma_{xy,y} + b_x)$

where right hand side is assumed to be constant in z-direction.
By integration:

$\sigma_{xz}(z) - \sigma_{xz}(s) = -(\sigma_{xx,x} + \sigma_{xy,y} + b_x)(\mathrm{z} - z_s)$

Since, $\sigma_{xz}(s) = 0$, as ice surface is a free surface.

$\sigma_{xz}(z) = (\sigma_{xx,x} + \sigma_{xy,y} + b_x)(z_s - z)$

On differentiation we get

$\sigma_{xz,x} = (\sigma_{xx,xx} + \sigma_{xy,yx} + b_{x,x})(z_s - z)$

$\sigma_{xz,y} = (\sigma_{xx,xy} + \sigma_{xy,yy} + b_{x,y})(z_s - z)$

As we got vertically down by ΔZ , there is a change in all 3 coordinates x,y,z.
So, σ_{xz} (ΔZ) = $\sigma_{xz,x}$ (Δx)+ $\sigma_{xz,y}$(Δy)+ $\sigma_{xz,z}$(Δz)

Equations of force balance in y-direction is:

$\sigma_{yx,y} + \sigma_{yy,y} + \sigma_{yz,z} + b_y$=0

Derivatives of σ_{yz} with x and y direction are calculated in the same way as done For σ_{xz} . Similarly , σ_{yz} at position Z is calculated like for σ_{xz}.

Derivatives with respect to x & y directions for stresses $\sigma_{xx}, \sigma_{yy}, \sigma_{xy}$ are assumed to be constant with z direction

Equation of force balance in z-direction are:

$\sigma_{zx,x} + \sigma_{zy,y} + \sigma_{zz,z} + b_z$=0

First two terms of the equation can be neglected

$$\sigma_{zz,z} = -b_z$$

$$\sigma_{zz}(b) = -b_z(z_b - z_s)$$

where b refers to bedrock and s refers to surface of ice

At the top surface stresses are

1) $\tau_{xx}, \tau_{yy}, \tau_{zz}, \tau_{xy}$ calculated from given velocity field

$$\tau_{ij} = B\dot{\epsilon}_e^{\frac{1}{n}-1}\dot{\epsilon}_{ij}$$

$$\sigma_{zz} = 0 = \sigma_m + \tau_{zz}$$

$$\sigma_m = -\tau_{zz}$$

$$\sigma_{xx} = \tau_{xx} + \sigma_m = \tau_{xx} - \tau_{zz}$$

$$\sigma_{yy} = \tau_{yy} - \tau_{zz}$$

2) $\tau_{xz} = 0, \tau_{yz} = 0$ Free boundary condition

3) Stress gradients are calculated by

$$\tau_{ij,k} = B\dot{\epsilon}_e^{\frac{1}{n}-1}\dot{\epsilon}_{ij,k} + B\left(\frac{1}{n} - 1\right)\dot{\epsilon}_e^{\frac{1}{n}-2}\,\dot{\epsilon}_{e,k}\dot{\epsilon}_{ij}$$

$$\sigma_{zz,x} = 0 = \sigma_{m,x} + \tau_{zz,x};\ \sigma_{m,x} = -\tau_{zz,x}$$

$$\sigma_{zz,y} = 0 = \sigma_{m,y} + \tau_{zz,y};\ \sigma_{m,y} = -\tau_{zz,y}$$

$$\tau_{xz,x} = 0; \tau_{xz,y} = 0$$

$$\tau_{yz,x} = 0; \tau_{yz,y} = 0$$

All stress gradients at the top surface are calculated as above:

At depth z:
Stress gradients for $\tau_{xx}, \tau_{yy}, \tau_{zz}, \tau_{xy}$ stay same
Calculation of stress gradients for τ_{xz} & τ_{yz} are explained before
Stresses τ_{xz}, τ_{yz} and σ_{zz} at bedrock are calculated as explained before
Stresses $\tau_{xx}, \tau_{yy}, \ \tau_{xy}$ are same as at top surface.

So all stresses and stress gradients are known throughout the glacier subject to certain assumptions.

Transformation of stresses at bedrock to a coordinate system with x, y directions tangential to bedrock.

$$\sigma_{pq_{bedrock,cs}} = l_{pi} l_{qj} \sigma_{ij_{surface,\,cs}}$$

Where l_{pi} and l_{pq} are cosines of angles between bedrock axes p, q and surface axes i, j.

Also $\sigma_{pq,r_{bedrock,cs}} = l_{pi} l_{qj} l_{rk} \sigma_{ij,k_{surface,\,cs}}$

Basal drag = $\tau_{xz_{bedrock}}$

Lateral drag = $\tau_{xy,y}$ (volume of glacier over a square of 1m of bedrock)

LSG = $\sigma_{xx,x}$ (volume of glacier over a square of 1m of bedrock)

Driving stress = ρg (volume of glacier over a square of 1m of bedrock) l_{xZ}

At each point sum of forces due to basal drag, lateral drag, LSG and driving stress is Equal to zero because of equilibrium equations used. Above terms are calculated for both Surface and bedrock coordinate systems.

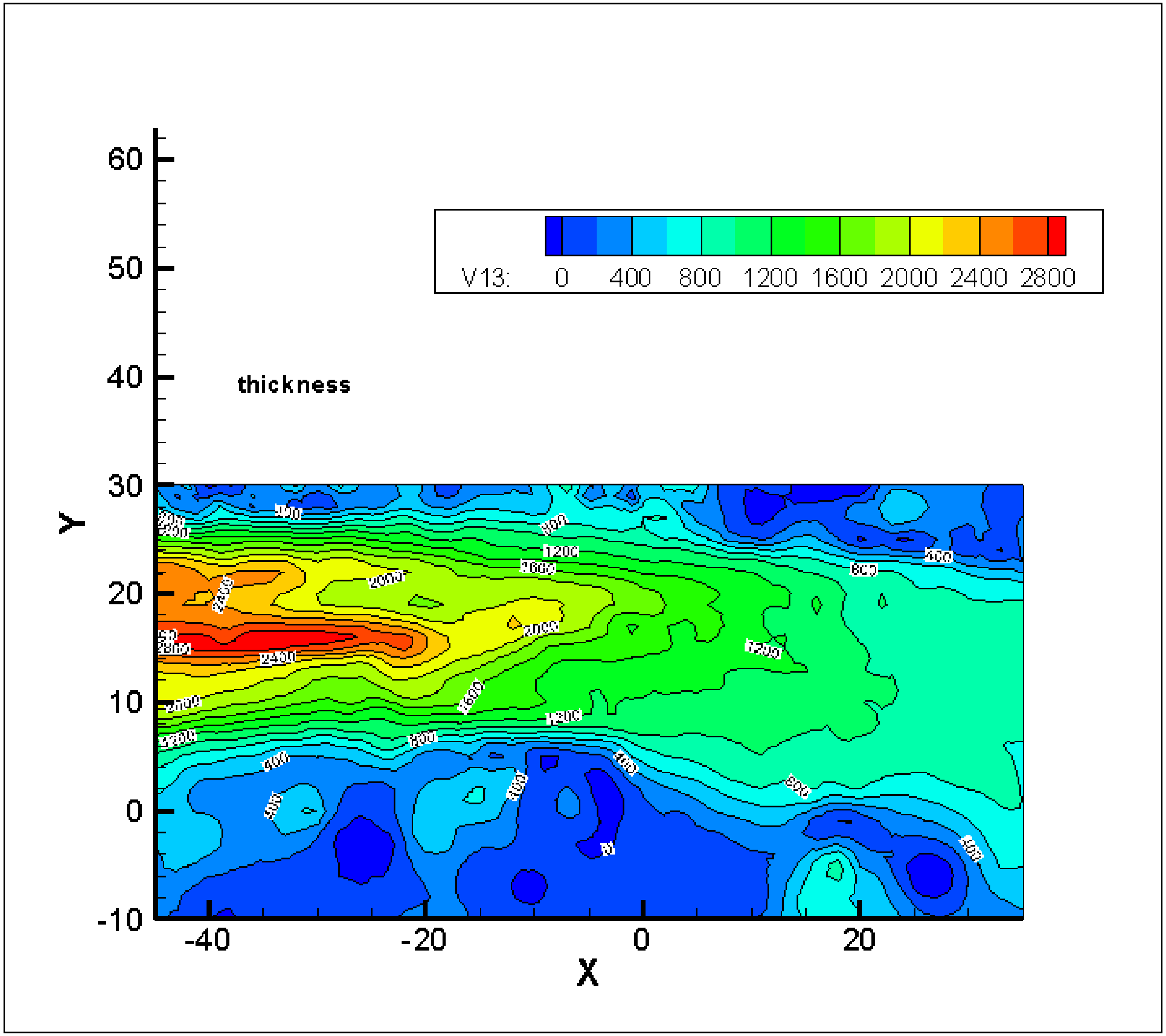


Fig. 2 Ice thickness of glacier.

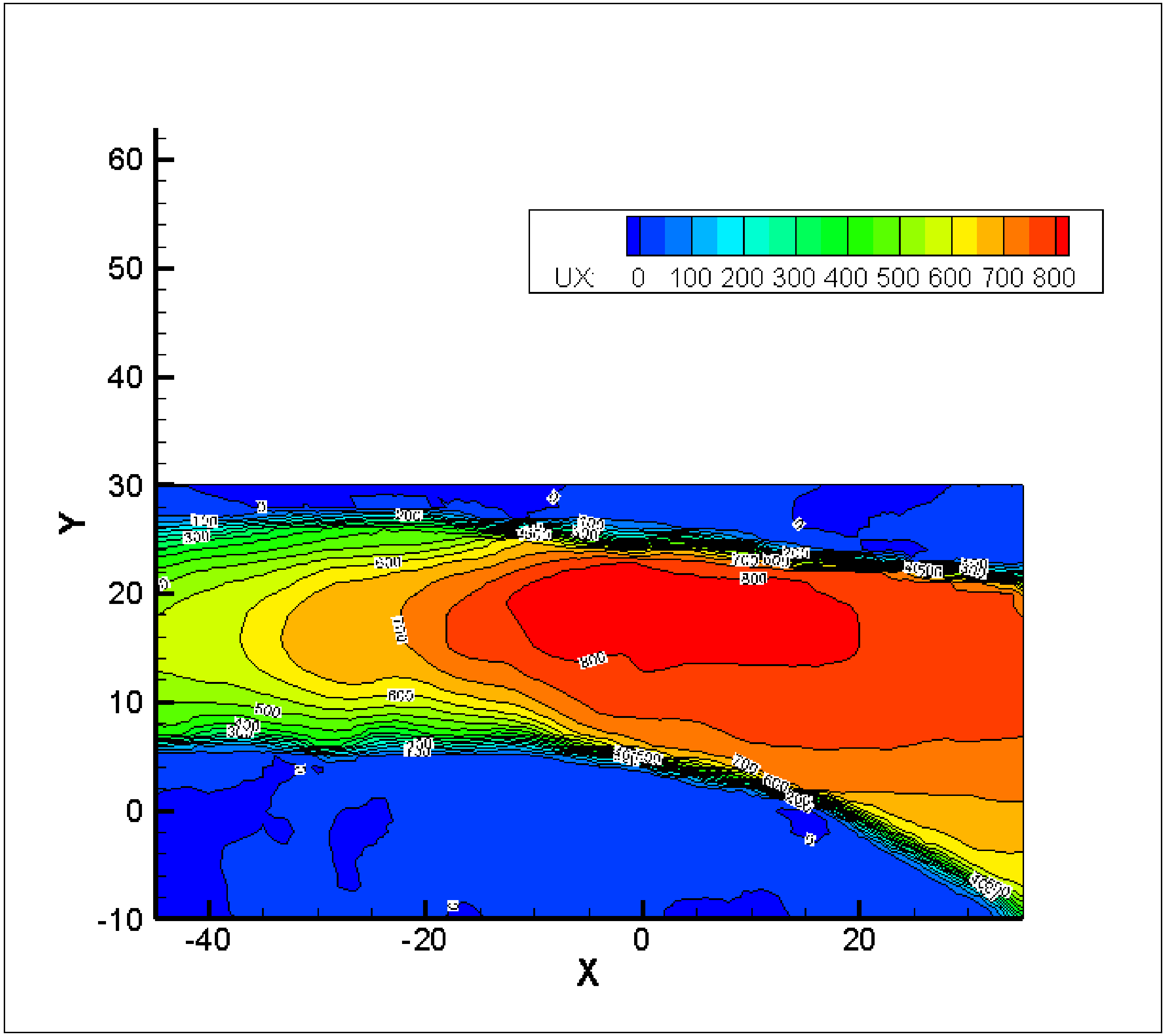


Fig. 3 Surface velocity.

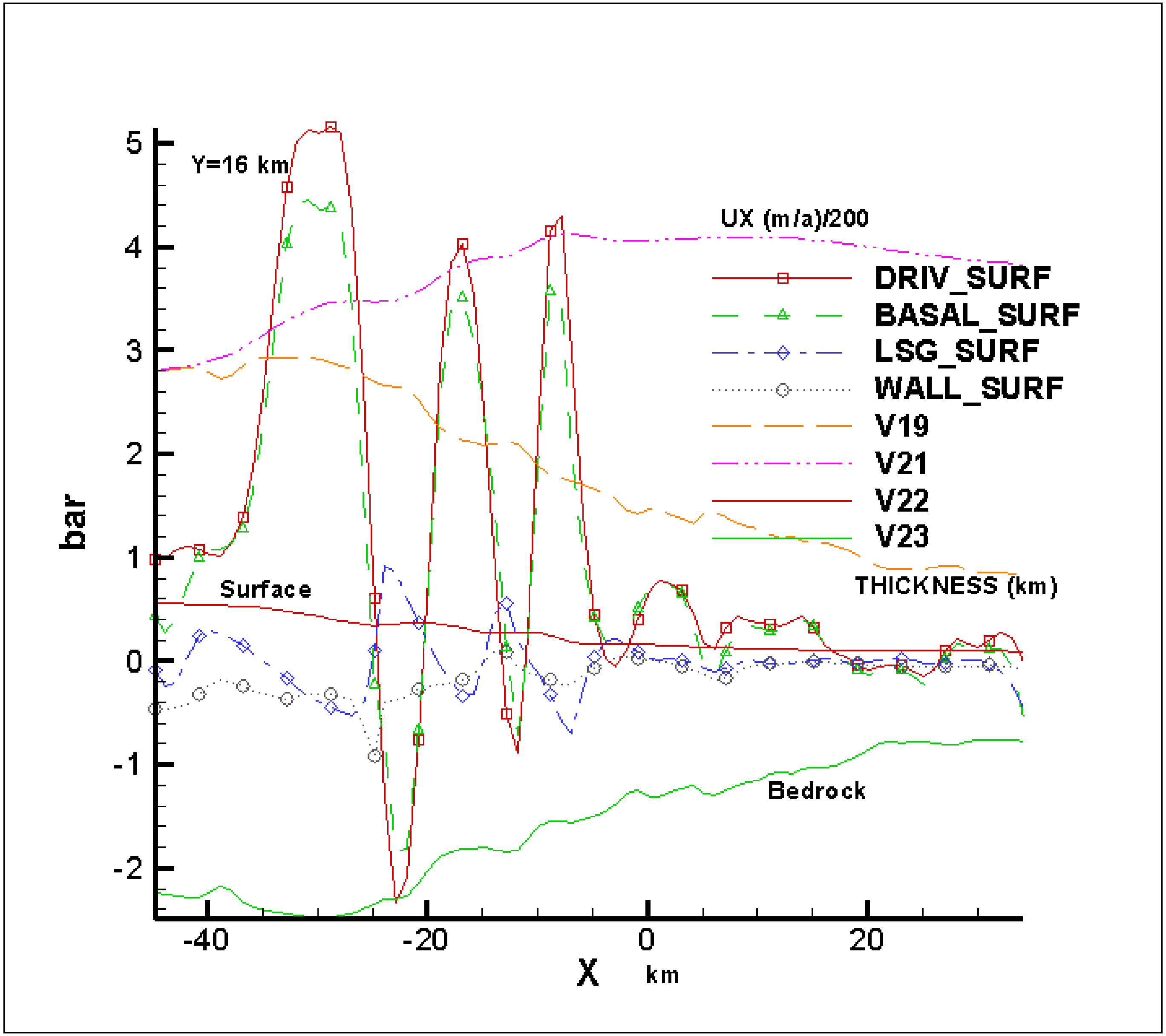


Fig. 4 Forces along center line using surface coordinate system.

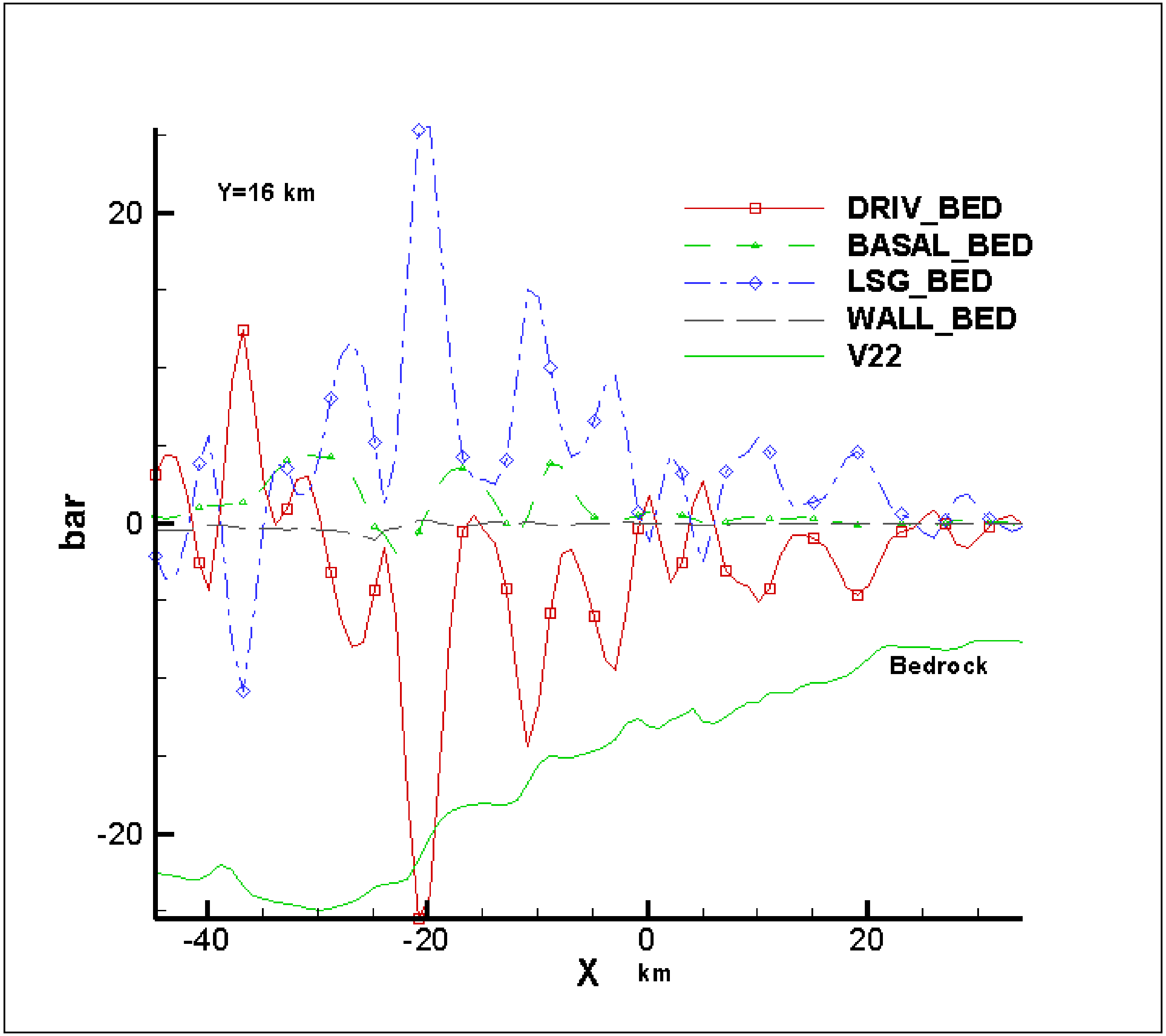


Fig. 5 Forces along center line using bed coordinate system.

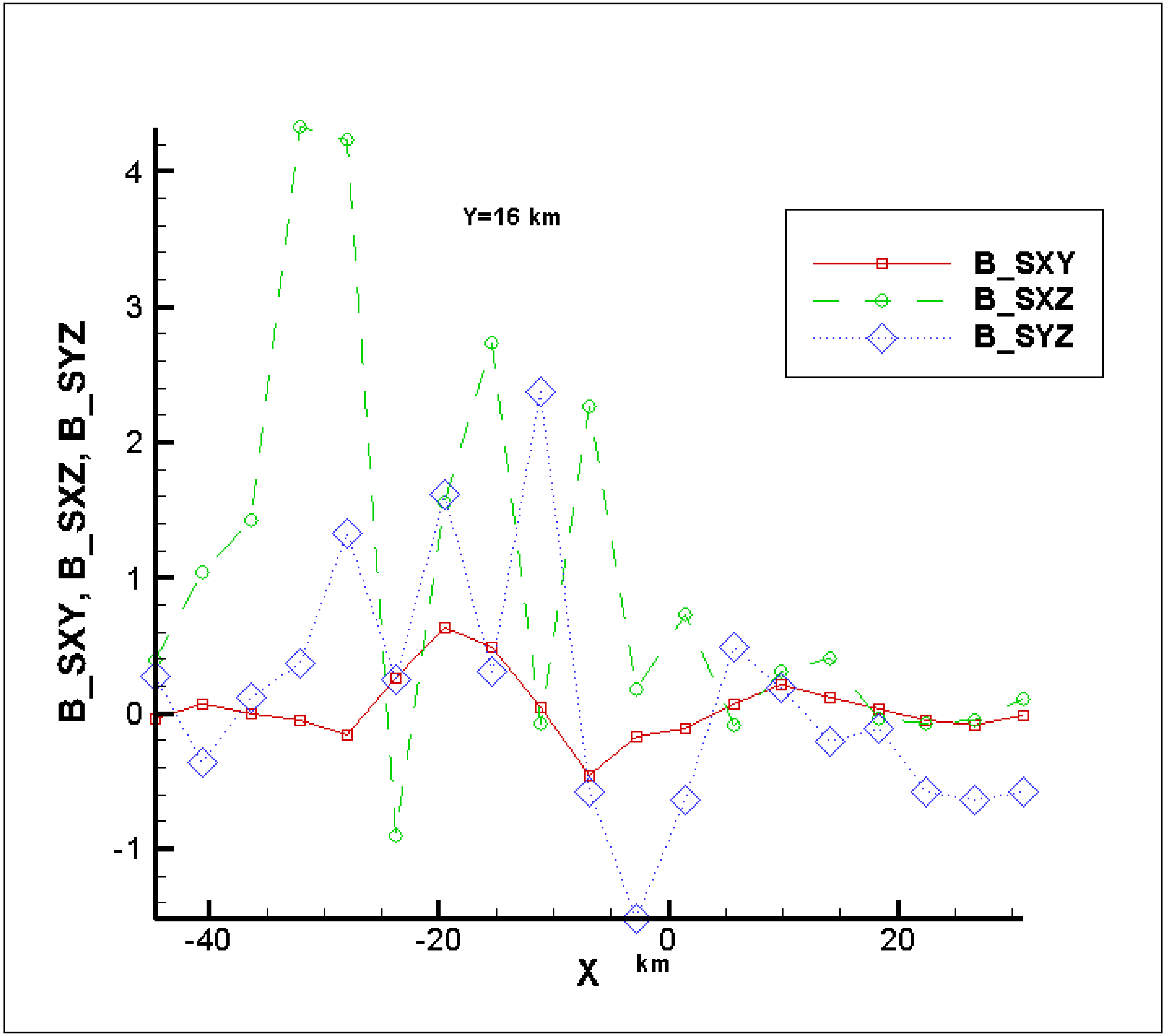


Fig. 6 Stresses at bedrock along center line for bedrock coordinate system.

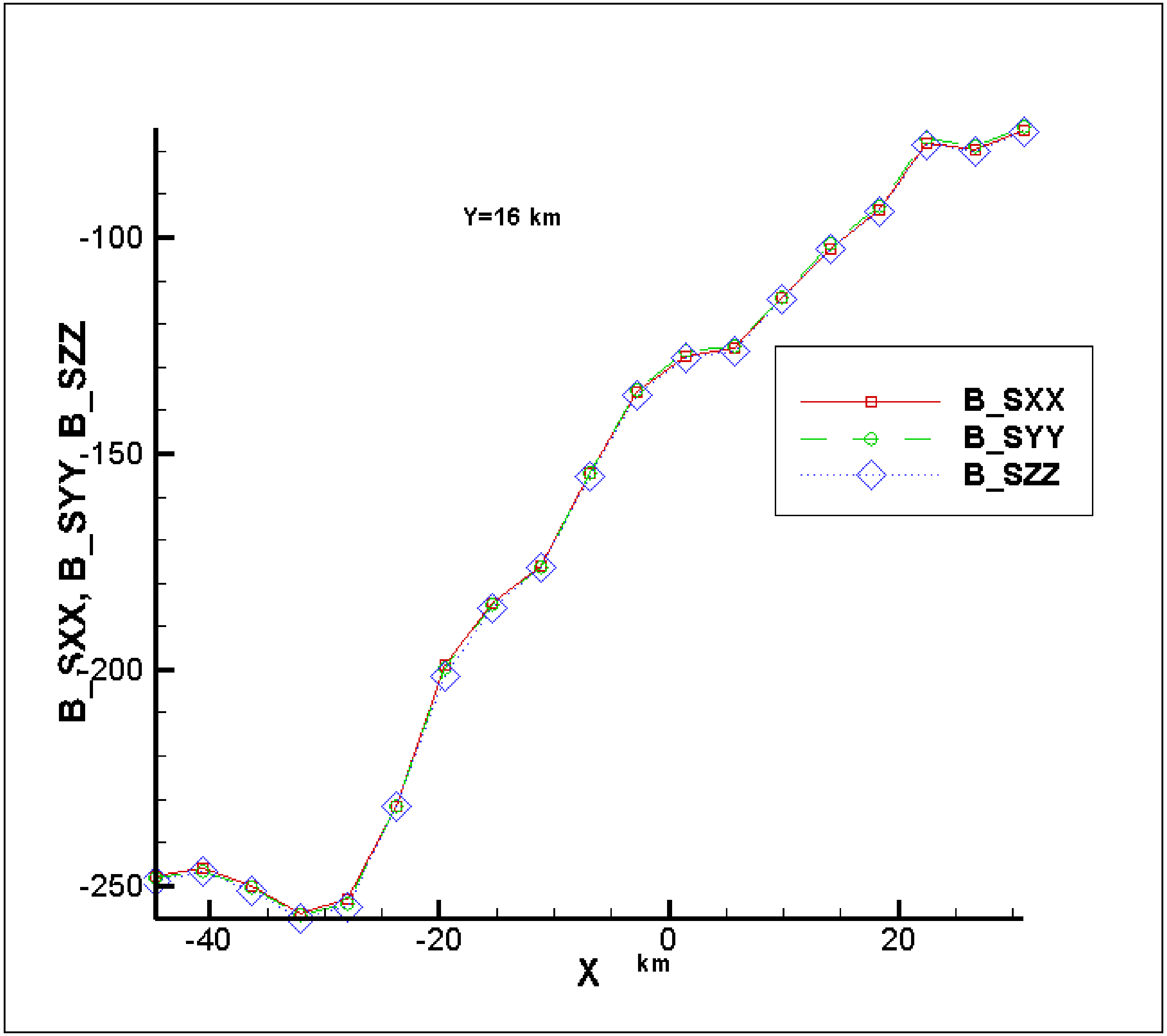


Fig. 7 Stresses at bedrock along center line for bedrock coordinate system.

(a)

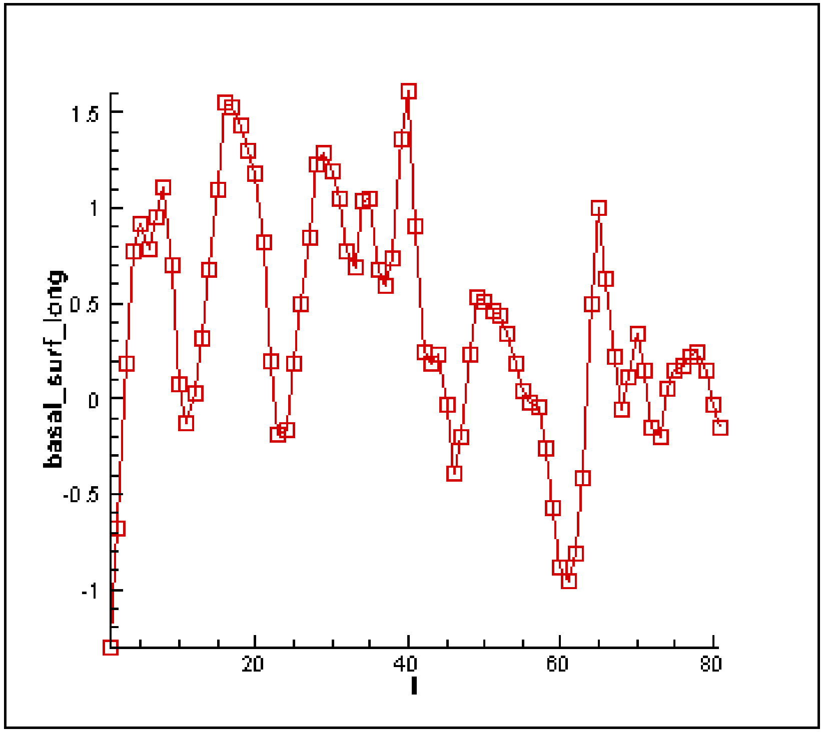
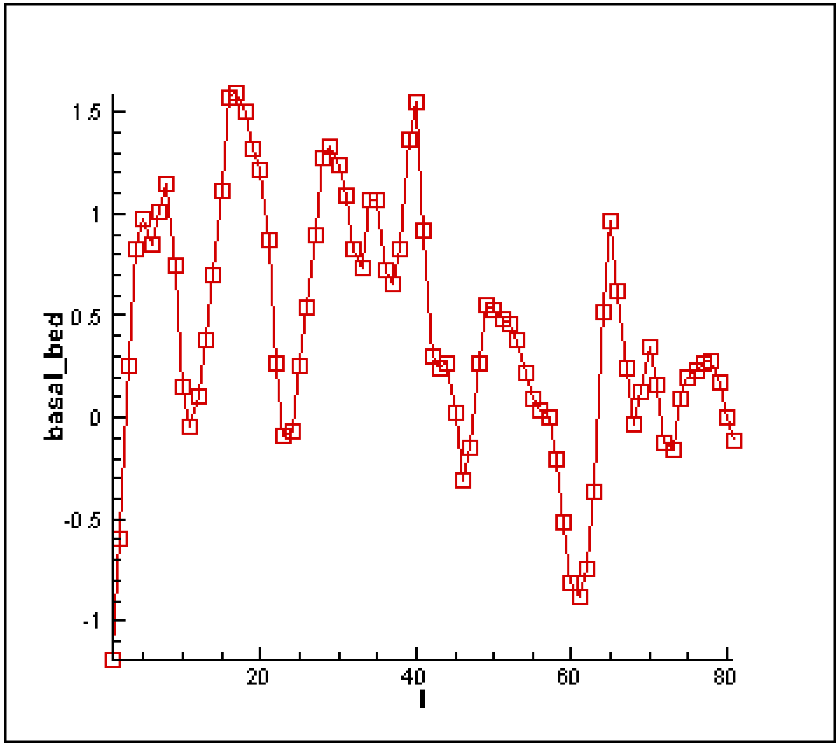
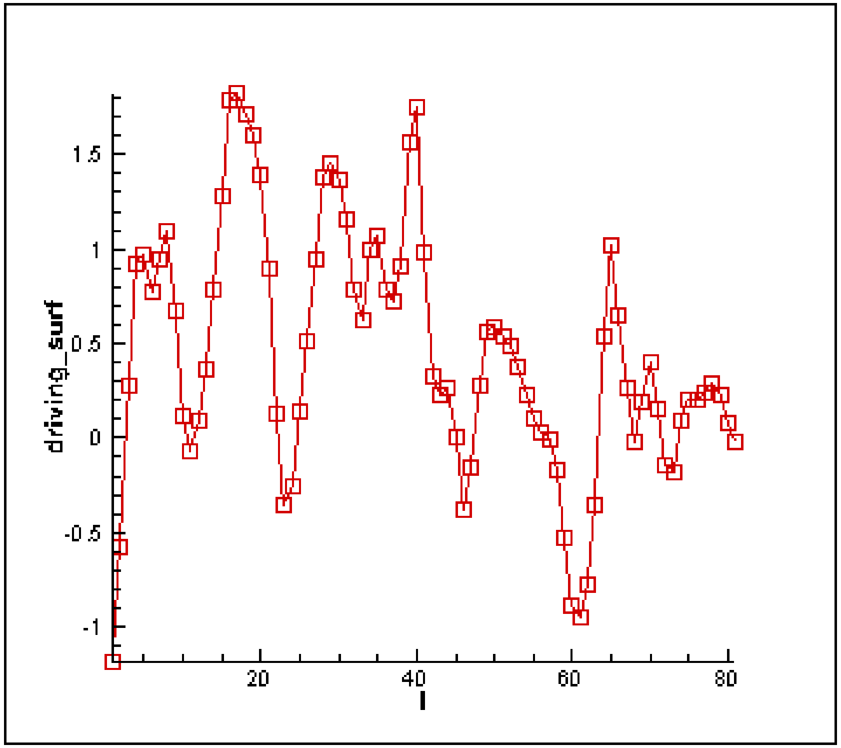


Fig. 8 Width wise average basal drag along the length of glacier (a) wrt surface CS (b) wrt bed CS.

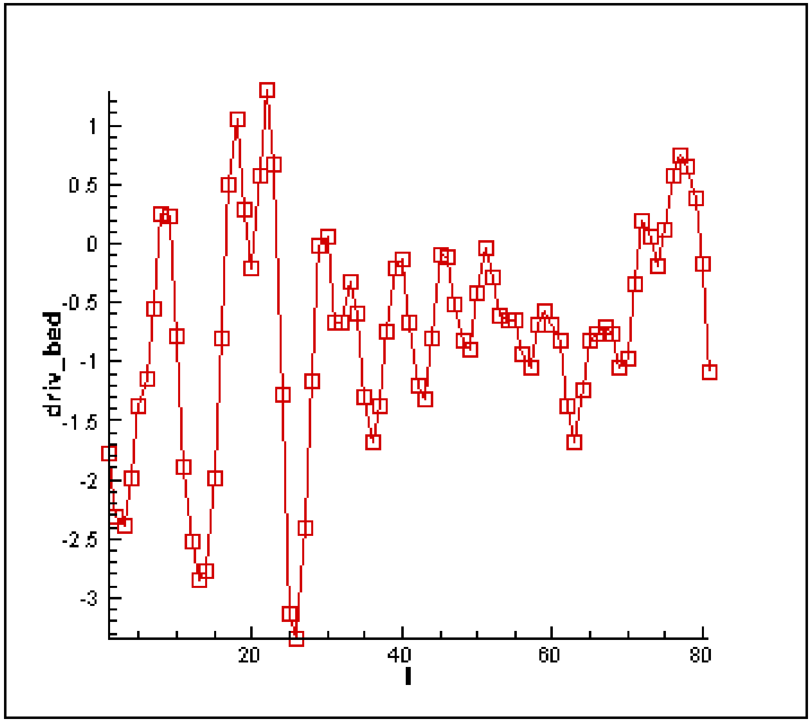
(b)



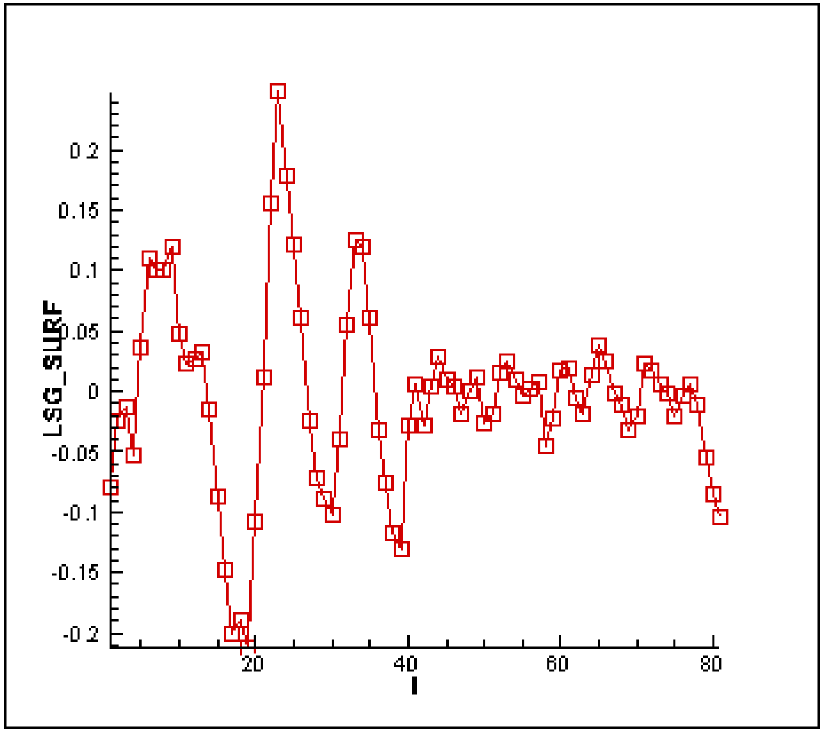


(a)

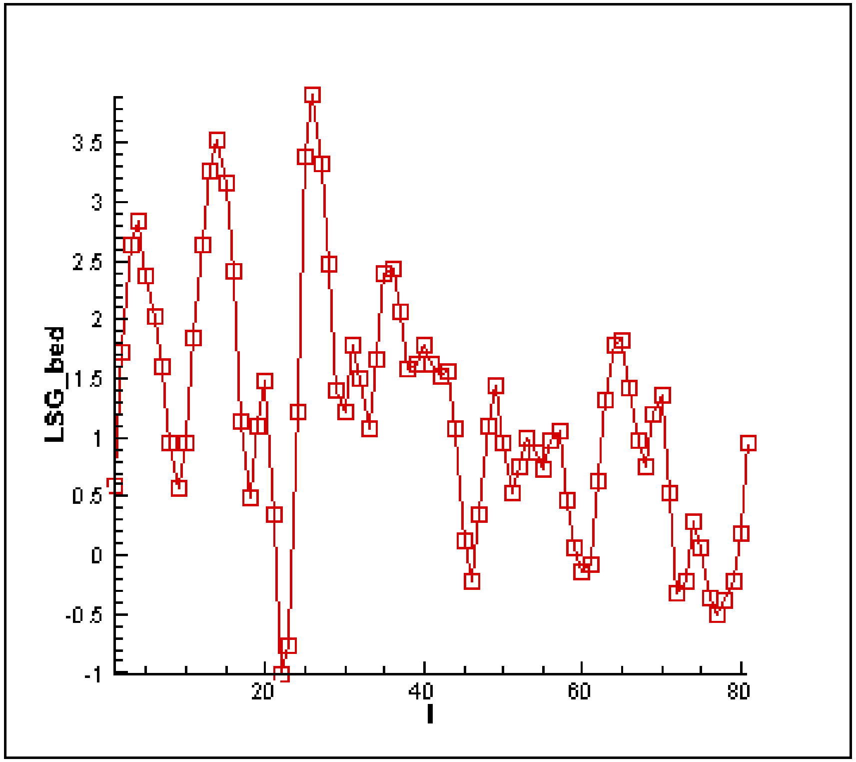
Fig. 9 Width wise average of driving force along the length of glacier (a) wrt surface CS (b) wrt bed CS.



(b)

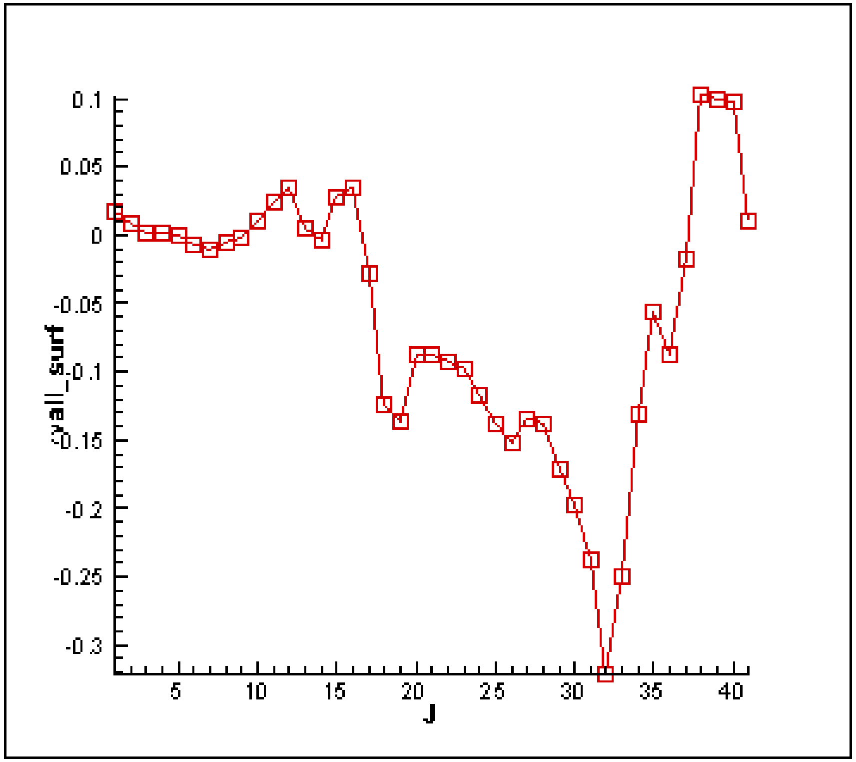


(a)

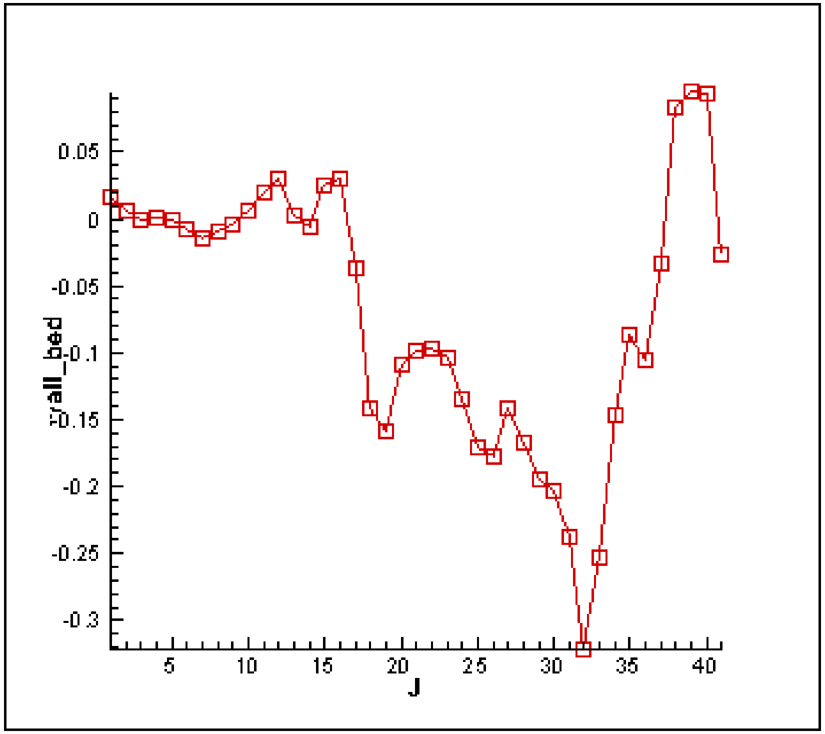


(b)

Fig. 10 Width wise average LSG along the length of glacier (a) wrt surface CS (b) wrt bed CS.



(a)



(b)

Fig. 11 Length wise average lateral drag along the width of glacier (a) wrt surface CS (b) wrt bed CS.

Conclusions

- Stress analysis is performed for Byrd glacier, Antarctica using data on surface velocity and elevation of surface and bed.
- For each data point, calculations are made using coordinate system with coordinate directions tangential and normal to glacier surface.
- Stresses and stress gradients are further transformed using stress transformation rules for the coordinate system with coordinate directions tangential and normal to bed surface.
- For Byrd glacier, average slope for surface and bed have opposite signs, making gravitational force opposing flow when calculated using bed slope.
- Force calculations with respect to the bedrock-coordinate-system show that longitudinal stress gradient provides the driving force that is balanced by gravitational force, basal resistance and wall shear force.

- Acknowledgement

I thank Prof. C. J. Van Der Veen, University of Kansas, for sharing the data of Byrd glacier.

References

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