Fat tails and volatility clustering in experimental asset markets $*$

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Abstract

This paper presents results from experimental asset markets with asymmetric fundamental information. We observe leptokurtic returns and a slowly decaying autocorrelation function of absolute returns. In contrast to results from heterogeneous agent models (HAM's) we find that noise has no significant influence on the emergence of fat tails. Instead, we observe a significantly positive relationship between the degree of heterogeneity of fundamental information and absolute returns, which suggests that heterogeneous fundamental information is the source of fat tails. With respect to volatility clustering, we discover an intra-periodical pattern where absolute returns decrease after the arrival of new asymmetric fundamental information.

JEL-classification: C16, C91, D82, D83

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Keywords: Asymmetric information, experimental economics, fat tails, volatility clustering, stylized facts

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1 Introduction

In this paper we want to tackle the question of the origins of the leptokurtic distribution of returns and the volatility clustering property, the most common cited stylized facts in the literature.

The studies of Mandelbrot (1963a, b) and Mandelbrot/Taylor (1967) were the first to show that returns on financial markets are not Gaussian, but exhibit excess kurtosis ('fat tails'). This is supported by more recent work by Bouchaud/Potters (2001), Cont (1997, 2001), Dacorogna et al. (2001), Mantegna/Stanley (2000), Plerou et al. (1999), Rachev (2003), Voit (2003) and others. Mandelbrot suggests that returns in financial markets are non-Gaussian stable Levy-processes, which was later called Stable Paretian Hypothesis (Rachev 2003, p. ix). Press (1967) suggests compound Poisson processes for the variance parameter of normal distributions as reason for the emergence of fat tails and Clark (1973) claims that finite-variance distributions like the lognormal-normal distribution fit data in financial markets better than any stable regime. He explains his statement with the different evolution of price series on different days due to varying information. Trading may be slow on days when no information is available. When new information appears on the market, the price process evolves much faster.

Over the past ten years many different approaches have been developed to reproduce these stylized facts in artificial markets with heterogeneous agents, so called HAM's – heterogeneous agent models - (e.g. Arthur 1997, Brock/Hommes 1998, Brock/LeBaron 1996, Hommes 2002, Iori 2002, Kirman/Teyssière 2002, LiCalzi/Pellizzari 2003, Lux 1995, Lux 1998, Lux/Marchesi 1999, 2000, Raberto et al. 2001, and Youssefmir/Huberman 1997). In two seminal papers, Lux/Marchesi (1999, 2000) attribute volatility clustering and the emergence of fat-tailed returns mainly to the agents' switching between fundamentalist and chartist strategies. Other models (e.g. Youssefmir/Huberman 1997, and Brock/Hommes 1998) find similar results. Once a certain threshold value of chartists is exceeded, the system

becomes unstable and extreme returns occur. During these regimes prices deviate strongly from their fundamental values, creating bubbles or crashes. As a consequence, the fundamentalist strategy becomes more profitable, inducing more and more agents to switch from a chartist to a fundamentalist strategy. This switching behaviour slowly brings prices back towards the fundamental value and is the stabilizing device of the system, causing a slow decay in the autocorrelation function of absolute returns.

Plott/Sunder (1982) were the first one to report excess kurtosis and the lack of autocorrelation of returns in price data generated in an experimental market. However, they do not deliver an explanation of the observed properties.

Our experimental asset markets show that noise trading (trading not based on fundamentals) does not play a major role for the fat tail property of returns which stands in contrast to heterogeneous agent models (HAM's) that use a chartist/fundamentalist framework. Instead, heterogeneity of fundamental information is the driving force for trading, volatility, and ultimately the emergence of fat tails. Furthermore, in HAMs the slow decay of the autocorrelation function of absolute returns is usually caused by increasing numbers of chartists switching back to a fundamentalist strategy. Our results do not corroborate this, but we find an intra-periodical pattern of decreasing absolute returns after the arrival of new fundamental information. Again, noise does not play a role for the emergence of this stylized fact.

The paper is structured as follows: Section 2 presents the market model. Section 3 focuses on the empirical properties of the experimental markets. The discussion in section 4 explains the reasons for the empirical results in the experimental markets, and section 5 summarizes the main findings of our study.

2 Market model and experimental implementation

2.1 Model description

1

The main innovation of our market model is the asymmetric information structure of traders. In the above-mentioned agent-based models traders are usually endowed with identical fundamental information. In theoretical models with asymmetric fundamental information (e.g. Grossman/Stiglitz 1980, Figlewski 1982, Hellwig 1982, Kyle 1985, 1989) traders are usually divided into two groups: insiders and uninformed traders. In this paper we extend this approach by introducing more than two information levels. Our model includes nine (treatment 1) and five (treatment 2) different information levels, ranging from almost uninformed traders through average informed traders to insiders.

Specifically, trader I_i knows the dividend of this and the future $(i-1)$ periods. Thus, a trader with information level I_1 knows the dividend of the current period; I_2 knows the dividend of the current and the next period, and so on. At the end of each period the respective dividend is paid out and deleted from the screen. At the start of the next period, each trader receives dividend information formerly only known to the next best informed. At the same time, a new dividend is generated and displayed to the best informed (the "insider"). This means, for example, that the former dividend of period (*k*+1) becomes the dividend of period *k* one period later.

For the sake of simplicity we assume that traders know the exact value of future dividends and no trader ever gets wrong dividend information. The dividend process is a random walk without drift, with D_k representing the dividend in period k. ε is a random term with an expected value of zero.¹

$$
D_k = D_{k-1} + \varepsilon \tag{1}
$$

¹ See Appendix for plots of the dividend process and the corresponding average price per period for all markets.

In addition to future dividends, we provide each trader with the conditional present value of the stock (based on his information), which is calculated using Gordon's formula:

$$
E(V | I_{j,k}) = \sum_{i=k}^{k+j-2} \frac{D_i}{(1+r_e)^{i-k}} + \frac{D_{k+j-1}}{(1+r_e)^{j-2} \cdot r_e}
$$
 (2)

 $E(V | I_{i,k})$ denotes the conditional expected value of the asset in period *k*, *j* represents the index for the information level and r_e is the risk adjusted interest rate.² We can see from equation (2) that the last dividend known to trader *j* is assumed to remain constant for an infinite number of periods. This is a consequence of the random walk dividend process. All the other dividends are also discounted using r_e . Figure 1 shows the resulting paths of the conditional expected values in treatment 1 as a function of time (measured in trading periods). 3

[Figure 1 about here]

Beginning with I_9 , the functions in Figure 1 are shifted for each information level I_i by (9*-j*) periods to the right. Information on the intrinsic value of the company that trader *I9*

² See Table 1 for specific numbers in the two treatments. We provide participants with r_e , as we are not interested in their risk preferences but in trading behaviour and the impact of the sequential arrival of fundamental information.

³ As these conditional expected values are the main fundamental information and are the most important benchmark for our analysis, we could have skipped the introduction of dividends and could have directly introduced an intrinsic value process that is shifted for each information level I_i by (9-*j*) or (5-*j*) to the right. We use dividends as a kind of 'cover story'. Our participants are business students, who are familiar with the dividend discount model. So, we ensured that they understood the concept of fundamental value, or in this case conditional expected value prior to the experiment.

receives in a certain period is seen by trader I_8 one period later, and by trader I_1 eight periods later, giving the better informed an informational advantage of time.⁴

This way of modelling the information structure is inspired by market reality, as in realworld markets relevant fundamental information is first known to insiders. Other major groups of investors, e.g. funds managers and large stake holders, have secondary access to information. This process of dispersion continues until information finally becomes publicly available through newspapers, TV and other media. So, information "trickles down" the market from the best informed to the broad public. Basically, all traders receive the same information – just at different times. The insider has an advantage of timing. This is supported by research on insider trading, which shows that insider information is superior and can generate above-average returns (e.g. Lakonishok/Lee 2001, Lin/Howe 1990, Krahnen/Rieck/Theissen 1999, and Jeng/Metrick/Zeckhauser 2003).

The trading mechanism is a continuous double auction with open order book (see trading screens in the appendix). In general, there are no limitations to trading, meaning that traders are allowed to freely place limit and market orders. However, traders are restricted from shorting both stocks and cash.

2.2 Experimental implementation

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In both treatments each trader is initially endowed with 1,600 Taler (experimental currency) in cash and 40 stocks of the risky asset. For holding the stocks they get dividends and for holding cash they receive the risk-free interest rate, r_f , at the end of each period.⁵ The total supply of stocks is fixed (40x9=360 in treatment 1, and 40x20=800 in treatment 2).

⁴ All other information like price paths, order books, etc. is identical for all traders.

 $⁵$ See Table 1 for specific numbers used in the two treatments.</sup>

During the experiment traders are continuously informed on their cash and stock holdings and on their wealth, which is calculated as the sum of the cash holdings and the stock holdings (number of shares held multiplied by the current stock price). Prices of all previous transactions in the current period are displayed to all traders together with the mean prices of all previous periods.

This paper is based on two treatments: Treatment 1 consists of nine information levels and one trader per information level (individual markets are denoted by 9_Mx^6). Treatment 2 involves five information levels and four traders per information level, hence a total of 20 traders per market (consequently called 20_Mx markets).

In treatment 1 we set up 30 periods of trading, each period lasting for 100 seconds. In treatment 2, each experimental market was randomly terminated between periods 20 and 30 with equal probability to avoid strategic behaviour of participants in the last period. Trading time was again 100 seconds per period.

At the beginning of each experimental session, traders were briefed with identical written instructions that took about 20 minutes to go through.⁷ Three trial periods in treatment 1 and four trial periods in treatment 2 followed to familiarize participants with the trading screen.⁸

In treatment 1, traders were paid according to their average wealth in the last period that was benchmarked by the average wealth of all traders to ensure that manipulating the price

 $6 \times$ 'x' always indicates the number of the market in the corresponding treatment.

 $⁷$ See experimental instructions in the appendix.</sup>

⁸ However, after the experiment we realized that in periods 1 and 2 of some markets of treatment 1, several traders were still too unaccustomed in handling the trading surface, which caused some mistakes in trading (e.g. selling at a low price when instead buying was intended). To limit this influence on our results we excluded the first two periods from the analysis. Due to an improved introduction and four trial periods, we observed no such irregularities in treatment 2.

towards the end of the last period would be useless.⁹ In treatment 2 benchmarking was based on period-by-period trading success. As a consequence, no manipulation was detected here. The experiments were conducted at the University of Innsbruck with business students. Each session (market) lasted for about 80 minutes with an average payment of ϵ 18. Treatment 1 consisted of 6 markets, treatment 2 of 5 markets. In total, the 154 participants initiated 8,563 trades. This large data set should lead to a high reliability of our results. The experiment was programmed and run with z-Tree (Fischbacher (1999)). Table 1 summarizes the differences between the two treatments.

[Table 1 about here]

3 Empirical properties of the markets

1

3.1 Scaling of the distribution of price fluctuations

We start with some basic statistics of the trading data. The returns presented are computed discretely following equation (3):

$$
R_{t} = \frac{P_{t} - P_{t-1}}{P_{t-1}}
$$
\n(3)

with R_t being the return, *t* indicating tick and P_t denoting the price at tick t .¹⁰

⁹ Nevertheless, we observed manipulation towards the end of the last period in at least two markets. So, we decided to remove the last two periods from our analysis as well. Consequently, all the data presented for treatment 1 is from 26 trading periods.

¹⁰ As we have a multi-period model, inter-period returns are included in the data we use for our analyses. However, to examine any possible influence of inter-period returns we repeated our analyses and tests by excluding these returns. The results were practically unchanged. As an example we include the data for

Table 2 indicates that all markets exhibit excess kurtosis, similar to real financial markets. Column "*T*" shows the number of return observations, calculated from $(T+1)$ transactions per market.

[Table 2 about here]

To help distinguish the excess kurtosis stemming from the market design from that caused by other factors discussed later, we computed a 'benchmark excess kurtosis'. For each individual transaction that took place in the market we calculated a 'fundamental price' (average of the conditional expected values (see equation (2) of the two participating traders) and calculated returns as if transactions took place at these prices. From the resulting returns we calculated the 'benchmark excess kurtosis' presented in the sixth column of Table 2. This would have been the excess kurtosis if all traders had traded on their fundamental information provided. The actual kurtosis resulting from real transaction prices is higher than this 'benchmark excess kurtosis' in ten of eleven markets.

Computing the Kolmogorov-Smirnov test statistics on gaussianity, we had to reject the null hypothesis in all eleven markets $(p<0.001)$. As an example, Figure 2 shows the cumulative distribution function (CDF) of absolute returns $|R_t|$ for market 9_M1 and for the NYSE composite index with daily data from January 1st 1997 to December $31st$ 2000 ($n=1009$) trading days). This demonstrates that our markets show properties similar to real financial markets¹¹

[Figure 2 about here]

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excess kurtosis without inter-period returns in the seventh column of Table 2. In further analyses we only present data including inter-period returns.

 11 Note that the other ten markets show very similar properties.

However, kurtosis is an ambiguous concept for measuring the fat tail characteristic. As a complement, exponents of empirical power laws have been proposed as measures for the 'fatness' of tails. Using an estimator suggested by Hill (1975), the Pareto exponent of the tails can be computed. In the applied economics literature, it is common practice to calculate the exponent for the 10%, 5% and 2.5% tail. To do this the elements of a return series have to be put in descending order and the last *x*% are selected as the "*x*% tail". Equation (4) presents the formula for computing the Hill estimator with *m* being the number of observations located in the corresponding tail of the distribution and *n* representing the total number of returns.

$$
\alpha = \frac{1}{\sum_{i=1}^{m} [\ln(R_{n-i+1}) - \ln(R_{n-m})]} \tag{4}
$$

As can be seen in Table 3, the medians of both treatments exhibit similar tails as real world markets, where Hill estimators range from approximately 2 to 6 with lower values denoting fatter tails (Voit 2003).

[Table 3 about here]

3.2 Volatility clustering

Volatility clustering is another well-known stylized fact in financial markets (e.g., Bouchaud/Potters 2001, Cont 1997, 2001, Mantegna/Stanley 2000, Plerou et al. 1999, Voit 2003). In simple terms, volatility clustering manifests itself as periods of tranquillity interrupted by periods of turbulence. The change between these two extreme regimes is a slow process so that large returns slowly decline until a relatively tranquil state is reached. Figure 3 illustrates this typical pattern for market 20_M3. Turbulent phases with large price changes alternate with relatively silent phases of small price activity.

[Figure 3 about here]

By the same token it is well-known that price movements do not exhibit any significant autocorrelation. Thus, we see a rapid decay of the autocorrelation function of price changes. There is also agreement that the absence of autocorrelation in returns does not imply the independence of the increments. Simple nonlinear functions of returns, such as squared returns or absolute returns, show significant positive autocorrelation. Figure 4 compares the autocorrelation function of returns with the autocorrelation function of absolute returns, which shows strong persistence. Especially in the larger and more competitive markets of treatment 2, this long-range dependence is much more pronounced due to higher trading activity.¹²

[Figure 4 about here]

4 Causes for fat tails and volatility clustering

4.1 Scaling of the distribution of price fluctuations

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Plott/Sunder (1982) were the first to report the emergence of fat tails and a lack of autocorrelations in returns in experimental markets but do not deliver an explanation. Their suspicion that fat tails emerged especially when prices are far from equilibrium, was not supported by their experimental data.

 12 See appendix for plots of the autocorrelation function of returns and absolute returns for all markets.

Our data does not allow us to differentiate among various mathematical explanations proposed in the 1960s and 1970s (e.g. Clark 1973, Mandelbrot 1963a, b, Mandelbrot/Taylor 1967, and Press 1967).

However, we can contribute to a strand of literature driven by different heterogeneous agent models. Several researchers (e.g. Brock/Hommes 1998, Cont/Bouchaud 2000, Gaunersdorfer/Hommes 2000, Levy/Levy/Solomon 2000, LiCalzi/Pellizzari 2003, Lux 1998, Lux/Marchesi 1999, 2000, Raberto et al. 2001) propose artificial markets with agents that use very simple trading strategies. These markets have the advantage of being simple enough to allow an analysis of the relation between trading behaviour and market properties. Especially the models of Brock/Hommes (1998), Gaunersdorfer/Hommes (2000), Lux (1998), Lux/Marchesi (1999, 2000), and Youssefmir/Huberman (1997) have similar origins for the occurrence of fat tails and volatility clustering. As long as their markets are near equilibrium, meaning that prices are close to the fundamental values, only small price deviations occur. But due to herding behaviour of the computerized agents, sometimes a burst of activity occurs and large price deviations result. In a first step we will test whether the fat tail property in our model roots in the same cause, or whether we have to look for other explanations.

Conjecture 1: The fraction of trades conforming to the fundamental strategy is negatively related to absolute returns.

To operationalize the terms 'fundamentalist strategy' or 'fundamental trading', we use the fundamentalist/chartist framework of Lux (1998) and Lux/Marchesi (1999, 2000), who state that *"the fundamentalist strategy […] implies buying (selling) when prices are below (above) the fundamental value"*, Lux (1998, p. 151). In the models of Lux (1998) and Lux/Marchesi (1999, 2000) (100% minus percentage of 'fundamentalist strategy') equals a chartist strategy. In the markets presented here (100% minus percentage of 'fundamentalist strategy') equals noise trading, thus trading not based on fundamentals. So, due to the

impossibility to measure Chartism in experimental asset markets exactly, we replaced the terminus 'Chartism' by the terminus 'Noise', which means trading not based on fundamentals.

For every transaction we computed whether each trader acted according to her fundamental information or not by comparing her action to the action conforming to fundamental trading. In the case of a buy the following condition must be satisfied to label a transaction "fundamentalist":

$$
E\big(V \mid I_{j,k}\big) > P_t \tag{5}
$$

In case of selling, however, we must have that:

$$
E[V|I_{j,k}] < P_t \tag{6}
$$

We calculate the fraction of fundamentalist trades in the whole market as well as in the "tails" of the cumulative distribution (for 10%, 5% and 2.5% of extreme values). Table 4 shows that on average about 60% to 75% of all transactions are fundamentalist.

[Table 4 about here]

Qualitatively, we can see that there is no clear trend concerning the relationship of the fraction of fundamentalist trades and absolute returns. In markets 9_M1, 9_M2, 20_M3 and 20_M4 large absolute returns are negatively related to the degree of fundamental trading. Therefore, they coincide with a higher degree of noise. However, even when large absolute returns occur, more than half of the trades are fundamentalist. Thus, we cannot call this a noisy regime. In all other markets we cannot find the relationship proposed by Lux/Marchesi (1999, 2000). In some markets, like 9_M6 and 20_M1, the relationship is clearly positive, which indicates that large absolute returns occur while fundamental trading is the predominant strategy.

To test conjecture 1, we run a correlation analysis using the Spearman-Rho test statistics, which does not require gaussianity. First we computed the average absolute returns per period according to equation (7):

$$
\overline{|R_k|} = \frac{\sum_{i=1}^{n_k} |R_{i,k}|}{n_k} \tag{7}
$$

 n_k represents the number of observations in period *k* and $|R_{i,k}|$ indicates the absolute return at tick *i* of period *k*. Secondly, we calculated the fraction of fundamentalist trades for each period by classifying transactions according to equations (5) and (6).

[Table 5 about here]

Table 5 shows Spearman's correlation coefficient for each market with its corresponding significance level. We see that only five of eleven markets are in line with conjecture 1, stating that the relationship of the degree of fundamental strategy and absolute returns is negative. Only market 9_M1 yields a significant result. On the other hand, the correlation coefficients of six markets are positive, four of them significant at the 1 or 5 percent level. In these markets the "tail" of the cumulative distribution function of absolute returns is characterized by an above-average fraction of fundamentalist trades, which is in strong contradiction to the findings of Lux/Marchesi (1999, 2000).

To obtain more reliable results, we pooled the data and found no significant correlation for treatment 1. However, we observe a significant positive relationship in the pooled data of treatment 2. We conclude that noise trading is not positively connected to the fat tail property of returns in our markets. Actually, a negative relationship of noise and large returns can be observed in treatment 2.

As already mentioned, our experimental asset markets are different to the ones cited above because of asymmetric fundamental information. In a next step, we will test whether

this heterogeneity is the source of fat tails. This may be possible, as Wang (1993) finds a positive relationship between heterogeneity of information and price volatility which leads to higher returns when heterogeneity increases.

Conjecture 2: Average absolute returns per period are positively correlated with the degree of heterogeneity in fundamental information.

As dividends following a random walk fluctuate sometimes widely and remain rather stable at other times, periods alternate between relatively heterogeneous (e.g. periods 8 to 15 in treatment 1) and homogeneous beliefs (e.g. period 20 to 30 in treatment 1). During phases with relatively stable dividends a longer prediction horizon of better informed traders does not yield significantly different estimates of the asset value. In contrast, in periods with strongly fluctuating dividends, traders' estimations of asset value vary strongly. Especially in these periods we expect large returns due to the heterogeneity of fundamental information.

We operationalize the criterion "heterogeneity in fundamental information" by the standard deviation of conditional expected values $E(V|I_{ik})$ with *k* denoting period and *j* indicating information level:

$$
\sigma_k = \sqrt{\frac{\sum_{j=1}^J \left(E\left(V \mid I_{j,k}\right) - \overline{E\left(V \mid I_k\right)}\right)^2}{n}}
$$
\n(8)

Large values of σ_k indicate periods with relatively heterogeneous fundamental information, while small σ_k denote relatively homogeneous periods. Additionally, we calculate the average absolute returns per period using equation (7). This provides us with a pair of data for each period consisting of the measure of heterogeneity of fundamental information and the corresponding average absolute returns of this period.

To test conjecture 2 without assuming gaussianity of the data, we compute the Spearman-Rho test statistics for each market and for the pooled data set per treatment.

[Table 6 about here]

Table 6 shows that the correlation coefficient of markets 9_M1, 20_M3, and 9_M5 is significantly positive at the 1% and 5% level, respectively. More importantly, the analysis of the pooled data set of treatment 1 demonstrates a positive relationship significant at the 5% level, as does the analysis of the pooled data set of treatment 2. We conclude that heterogeneous fundamental information is a major source for the emergence of fat tails in our experimental markets. In fact it is evident that a higher degree of heterogeneity in the conditional expected values causes larger price deviations.

4.2 Volatility clustering

As outlined above, the main sign of the appearance of clustered volatility is the long lasting positive autocorrelation of absolute returns as shown in Figure 4. In HAMs this phenomenon is often attributed to traders' switching between chartist and fundamentalist strategies. However, building on the results above, we opt for another explanation. If heterogeneous information is the main source of fat tails and volatility, and if we assume that traders learn from observed prices (e.g. Smith 1982), we should see higher volatility after the arrival of new information, i.e. at the beginning of each period. At this time heterogeneity of expectations is highest and large returns should result. In the course of each period (absolute) returns should decline as traders learn from prices and orders. As a consequence, the market moves towards a partial equilibrium until new fundamental information is injected into the market at the start of the next period. Then, the same patterns should start again. In accordance with our suggestion we formulate the following hypothesis:

Conjecture 3: Absolute Returns are negatively correlated to time within one period.

To operationalize this, we divide each period into ten sub-periods of ten seconds each and compute the average absolute return for each of the ten fractions according to equation (9). *l* denotes time fraction {1,…,10} within a period.

$$
\left| \overline{R}_{i} \right| = \frac{\sum_{i=1}^{n_{i}} |R_{i,l}|}{n_{i}}
$$
\n(9)

Fraction 1 comprises seconds 1 to 10 within each period, fraction 2 seconds 11 to 20, etc. n_l denotes the number of observations within a time fraction *l* across all periods.

Figure 5 plots the average absolute returns $\overline{R_1}$ as a function of time fraction *l* for all markets for both treatments. We can see that average absolute returns decrease with increasing *l*. At the beginning of each period large returns occur. Subsequently, they decline rapidly up to time fraction 5 and then stabilize at a relatively low level.

[Figure 5 about here]

To test conjecture 3, we correlate average absolute returns $\overline{|R_i|}$ with time fraction *l* according to the Spearman-Rho test statistics. We obtain results that strongly support our hypothesis.

[Table 7 about here]

Table 7 indicates that the correlation coefficients of all but three markets are significantly or highly significantly negative. Only market 9_M3 yields a positive but not significant correlation. As a result, the autocorrelation function of absolute returns of this specific market

shows a fast decay towards zero (see plot in the appendix). Analyzing pooled data sets for both treatments confirms a negative correlation which is significant at the 1% level.

In the model of Lux/Marchesi (1999, 2000), switching to the fundamentalist strategy is a stabilizing device. In our model it is not the switching between strategies that leads to the slow decay of the autocorrelation function of absolute returns. Instead, an intra-periodical pattern of learning that lowers absolute returns in the course of each period seems to be the source of the volatility clustering property. Again noise does not play a role for producing this stylized fact.

5 Conclusion

In this paper we tested experimental asset markets with asymmetric fundamental information for empirically observed properties of financial markets, known as stylized facts. We focused on the fat-tail property of returns and on the phenomenon of volatility clustering. Although each of the experimental markets consisted of only nine or twenty traders and trading lasted for less than one hour, we found similar properties as in real financial markets. We discovered excess kurtosis of returns, fast decay of the autocorrelation function of returns, and slow decay of the autocorrelation function of absolute returns.

A limitation of our study is that we do not have the data to differentiate among various mathematical explanations proposed in the 1960s and 1970s to explain the emergence of fat tails. Our data do not allow differentiation between stable and non-stable processes as the sources of the known stylized facts. However, we can compare our results with popular heterogeneous agent models. In contradiction to literature on these models noise does not play a major role for the emergence of fat tails in our markets. Instead, we found a significant positive relationship between the degree of heterogeneity of fundamental information and absolute returns in both treatments. In our markets, heterogeneity of fundamental information is the main driving force for trading activity, volatility and ultimately the emergence of fat tails.

With respect to volatility clustering we discovered an intra-periodical pattern of decreasing absolute returns, which yields a long-lasting positive autocorrelation of absolute returns in both treatments. At the beginning of each period, new asymmetric fundamental information is injected into the market and large amplitudes of price changes occur. In the course of each period these turbulences decrease as traders learn from past prices and the order book. This leads to relatively stable prices until new information is injected again.

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Appendix

A1: Experimental instructions for treatment 1 (the following instructions are translated from German)

Dear Participant! We welcome you to this experimental session and we kindly ask you to refrain from talking to each other for the duration of the experiment.

Background of the experiment

This experiment is designed to replicate asset markets where 9 participants in a market can trade the stocks of an imaginary company for 30 consecutive periods (months). You can increase your wealth by trading, and at the end you will receive a cash payment depending on your wealth.

Characteristics of the market

Each trader is endowed with 1600 Taler (experimental currency) and with 40 stocks worth 40 at the beginning of the experiment. The only fundamental information you receive is the dividend of the stock (monthly dividend equals monthly profit). Changes of the dividend per period have an expected value of zero and will fluctuate randomly at maximum +/- 50%. The market is characterized by an asymmetric information distribution. Worst informed traders are informed only about the dividend of the current period, while better informed ones know the dividend of the company a few months ahead. The best informed trader knows the dividend of the current period and the dividends of the coming 8 periods.

At the end of each period (which lasts 100 seconds) you will receive the current dividend for each stock you own. A risk free interest rate of 0.1% is paid for the cash holdings in each period. The risk adjusted interest rate for evaluation of the stock equals 0.5%.

Trading

The trading mechanism is implemented as a double auction. This means that each trader can buy or sell stocks freely. Therefore you can enter as many bids and asks as you wish within the range of 0 and 200 (with at maximum one decimal place).

Calculation of the conditional expected value (present value, PV)

Generally, it is up to you on what kind of information you will trade and how you will evaluate the stock. If you use your fundamental information, you can see the present value (PV) of all future dividends (of course only those you can estimate on the basis of your information level) on the bottom left side of the trading screen. Your PV is derived using Gordon's well-known formula, discounting the known dividends and using the last one as a continuous, infinite stream which was also discounted as a company is basically designed for infinity.

$$
PV_{k} = \sum_{k=0}^{n-1} \frac{D_{k}}{1.005^{k}} + \frac{\frac{D_{n}}{0.005}}{1.005^{n-1}}
$$

with *n* denoting the last period

Example: The dividends of this and the next 2 periods are 0.191; 0.214; 0.202. So, the PV on basis of this information level is calculated as follows: $0.191 + 0.214/1.005 + 0.202/0.005/1.005 = 40.40$. This PV is shown on the bottom left side of the trading screen.

Wealth

Your wealth is the sum of your cash holdings and the product of your stock holdings multiplied with the current price. If you buy a stock, your cash holdings are reduced by the price you paid and at the same time your stock holdings are increased by one share. Generally, the current price on the market is used for evaluation of your wealth, so your wealth will change even if you have not participated in the last transaction. After expiration of each trading period (month) an interest rate of 0.1% per month is paid for the current cash holdings, and the dividends for your stocks are added to your cash.

Example: If you own 1600 in cash and 40 stocks with a price of 40 and the dividend equals 0.215 at the end of a period, your wealth increases from 3200 to 3210.2 (Hence, the increase in wealth consists of $+1.6$ for interest earnings (= $1600*0.001$) $+8.6$ for dividend earnings (= $40*0.215$)).

Important details

- Per period you can trade as much as you wish (of course, only within the boundaries of your cash and stock holdings). Negative cash holdings are not possible.
- Trading time per period is 100 seconds, the remaining time is displayed at the top right side of the trading screen.
- Your payment at the end of the experiment depends on your wealth in the last period.

A2: Experimental instructions for treatment 2 (the following instructions are translated from German)

Dear Participant! We welcome you to this experimental session and we kindly ask you to refrain from talking to each other for the duration of the experiment.

Background of the experiment

This experiment is designed to replicate asset markets where participants in a market can trade the stocks of an imaginary company for *k* consecutive period.

Characteristics of the market

Each trader is initially endowed with 1600 Taler (experimental currency) and 40 stocks. The only fundamental information you receive is the dividend of the stock (quarterly dividend equals quarterly profit of the company) which follows a random walk process without drift:

$$
D_k = D_{k-1} + \varepsilon
$$

 D_k denotes the dividend of period k and ε represents a normally distributed random variable with an expected value of zero and a standard deviation of 15 percent. This period's dividend is therefore the best estimate for next period's dividend. The market is characterized by asymmetric information. The worst informed trader knows only the dividend of the current period, while better informed traders can estimate the dividends of the companies a few periods into the future. At the end of each period (after 100 seconds), you will receive the current dividend for each stock you own. A risk-free interest rate of 0.5% is paid for the cash holdings in each period. The riskadjusted interest rate for valuation of the stock equals 2.0% per period.

Calculation of the conditional expected value (present value, PV)

Generally it is up to you on what kind of information you trade and how you evaluate the stock. If you want to use your fundamental information (expected future dividends) you can see the present value (PV) of all future dividends (of course only those you can estimate on the basis of your information level) on the bottom left side of the trading screen. Your PV is derived using Gordon's well-known formula, discounting the dividends you know with the risk adjusted interest rate of 2.0% and assuming the last one as a continuous, infinite stream which is also discounted. If you follow this information it makes sense to buy at a price that is lower than your PV and sell at a price that is higher than your PV.

$$
BW_k = \sum_{k=0}^{n-1} \frac{D_k}{1.02^k} + \frac{D_n}{1.02^{n-1}}
$$

n indicates the , last' dividend you know

Example: The dividends of this (k=0) and the next 2 periods are 0.791; 0.814; 0.802. The PV on the basis of this information level is calculated as follows: $0.791 + 0.814/1.02 + 0.802/0.02/1.02 = 40.23$. This PV on the basis of your information level is shown on the bottom left side of the trading screen.

Trading

The trading mechanism is implemented as a double auction. This means that each trader can buy or sell stocks. You can enter as many bids and asks within the price range of 0 and 200 (with a precision of one decimal place) as you wish. Additionally, you have to insert the quantity you want to trade (1 to 10 shares). A new offer to buy is only accepted if the sum of this and all your outstanding offers to buy (price

multiplied by the corresponding quantity) is not higher than your current cash holding. Otherwise a message box appears to inform you that the offer is not valid.

This check is made to avoid that your cash holdings drop below zero. A new offer to sell will be accepted if the sum of this and all your outstanding offers to sell is lower than your current stock holding. Otherwise a message box appears. This check is made to avoid that your stock holdings drop below zero.

Example: Your current cash holdings equal 600 Taler. Your outstanding offers to buy equal 532.5 Taler, containing one offer of 10 stocks at a price of 35 Taler and another offer of 5 stocks at a price of 36.5 Taler. So, the product of your new offer to buy (price multiplied with stocks) should not exceed 67.5 Taler.

Wealth

Your wealth is the sum of your cash holdings and the product of your stock holdings multiplied by the current price. If you buy a stock your cash holdings are reduced and at the same time your stock position increases by the quantity you traded. Generally, for evaluation of your wealth the current price on the market is used (marking-tomarket), so your wealth will change even if you have not participated in the last transaction. After expiration of each trading period (quarter) for the current cash holdings an interest rate of 0.5% per quarter is paid and the dividends for your stocks are added to your cash.

Example: If you own 1600 in cash and 35 stocks with a price of 50 that pays a dividend of 0.815 at the end of a period, your wealth increases from 3350 to 3386.53 (+8.0 interest earnings (1600x0.005), +28.53 dividend earnings (35x0.815)).

Important details

- The experiment will be randomly terminated between period 20 and 30 with equal probability for each period.
- Your pay-off at the end of the experiment depends on your relative performance in the market. Your wealth at the end of each period will be compared with the average wealth in the market at the same time. This relation is summed up across all periods. Generally, your pay-off will be above average if you can manage to 'outperform' the market. Note that depending on your information level, your pay-off will be calibrated.

A3: Plots

A3.1. Average prices (black solid line) and dividends (grey line with asterisks) as a function of time

Treatment 1

Treatment 2

A3.2. Autocorrelation function of absolute returns (solid line with asterisks) and autocorrelation function of returns (solid line). The dashed lines represent the 95% confidence level

¹³ Due to a , shorter' memory in the absolute returns based on fewer traders and so much lower market activity, we plotted 40 lags compared to 100 lags in treatment 2.

Tables and Figures

Figure 1. Conditional expected values as a function of period of the uneven information levels in treatment 114

¹⁴ We only plotted the uneven information levels for better visibility.

<u>.</u>

 15 If we assume the periods to represent months the respective risk free and risk adjusted interest rates were 1.21 and 6.17 percent p.a. These are quite realistic numbers for real markets after adjusting for inflation.

¹⁶ With this parameterization one period in our model can be interpreted as one quarter of a year in real world financial markets. The respective annual returns are 2.02 percent p.a. risk free and 8.24 percent p.a. for the risky asset.

Treatment	Mean	Standard deviation	Excess kurtosis	T	"Benchmark" excess kurtosis"	Excess kurtosis*	T^*
9 M1	0.005	0.112	67.10	827	3.93	73.52	802
9 M ₂	0.001	0.039	7.85	505	2.25	8.33	480
9 M3	0.001	0.050	14.15	275	1.66	13.86	250
9 M4	0.003	0.082	116.42	870	4.35	126.20	845
9 M ₅	0.004	0.090	7.03	717	2.07	7.41	693
9 M ₆	0.002	0.062	20.63	898	6.39	21.48	874

Table 2. Elementary statistics.

* excluding inter-period returns

* excluding inter-period returns

Figure 2. Log-Log-Plot of the empirical cumulative distribution function (dots) of absolute returns of market 9_M1 (left panel) and of NYSE composite index, daily data, from January 1st 1997 to December 31st 2000 (right panel). The horizontal axis shows absolute returns, the vertical axis its cumulative density. The solid line represents the gaussian regime with same mean and standard deviation. The probability of large absolute returns is much higher than would be expected by the gaussian distribution.

Treatment 1	10 % tail	5 % tail	2.5 % tail
9 M1	4.16	5.09	4.09
9 M ₂	5.26	5.69	6.95
9_M3	5.49	6.88	5.28
9 M4	5.05	5.68	6.35
9 M ₅	4.76	12.05	11.98
9 M ₆	3.38	3.81	5.92
Median	4.91	5.69	6.14
Treatment 2	10 % tail	5 % tail	2.5 % tail
20_M1	7.96	9.79	20.28
20_M2	4.79	8.07	8.34
20_M3	4.74	5.80	8.23
20_M4	4.81	6.62	11.23
20 M ₅	4.61	6.52	7.03

Table 3. Hill Estimator for the 10%, 5%, 2.5% tail of the cumulative distribution function of absolute returns.

Figure 3. Volatility Clustering: Returns R_t as a function of tick of market 20 M3 (left panel) and of NYSE composite index, daily data, from January 1st 1997 to December 31st 2000 (right panel).

Figure 4. Autocorrelation function of returns R_t (solid line) and absolute returns |R_t| (solid line with asterisks) of market 20_M2 (left panel) and of NYSE composite index, daily data, from January 1st 1997 to December 31st 2000 (right panel). The 95% confidence interval is represented by the dashed lines. The persistence in autocorrelation of absolute returns is a sign of volatility clustering.

Table 4. Fraction of trades conforming to fundamental strategy in the whole market (all returns) and in the "tails" of the CDF (for 10%, 5% and 2.5% of extreme values).

Treatment 1	All Returns	10% Hill	5% Hill	2.5% Hill
9_M1	80.8	65.5	66.7	68.2
9 M2	77.0	73.5	73.1	60.7
9 M3	71.8	73.2	73.3	68.8
9 M4	59.6	61.4	60.2	60.9
9 M ₅	75.3	76.0	78.4	71.1
9 M ₆	79.8	84.0	89.1	89.1
Median	74.1	72.3	73.5	69.8
Treatment 2	All Returns	10% Hill	5% Hill	2.5% Hill
20_M1	57.9	66.7	68.4	80.0
20_M2	71.6	69.7	75.0	75.0
20_M3	64.8	56.0	51.9	51.8
20_M4	61.5	59.8	57.3	56.3
20_M5	59.9	61.5	60.2	61.7
Median	63.1	62.7	62.6	65.0

Treatment 1	Spearman Rho	Significance (one sided)	n	
9 M1	$-0.354**$	0.038	26	
9 M ₂	$0.100**$	0.481	26	
9 M3	$-0.042**$	0.418	26	
9 M4	$0.372**$	0.031	26	
9 M ₅	$0.416**$	0.017	26	
9 M ₆	$0.398**$	0.022	26	
Pooled Data	$0.004**$	0.481	156	
** significant at a 1% level; * significant at a 5% level				

Table 5. Spearman correlation coefficients of absolute returns per period and the percentage of fundamentalist strategy per period.

Treatment 1	Spearman Rho	Significance (one sided)	n	
9 M1	$0.379**$	0.028	26	
9 M ₂	$0.108**$	0.300	26	
9 M3	$0.013**$	0.474	26	
9 M4	$-0.035**$	0.434	26	
9 M ₅	$0.644**$	0.000	26	
9 M ₆	$0.161**$	0.216	26	
Pooled Data	$0.184**$	0.011	156	
** significant at a 1% level; * significant at a 5% level				

Table 6. Spearman correlation coefficients of the standard deviation of conditional expected values and average absolute returns.

Figure 5. Average absolute return as a function of time fraction l of treatment 1 (left panel) and treatment 2 (right panel).

Table 7. Spearman correlation coefficients of average absolute returns and time fraction l for both treatments.

Treatment 1	Spearman	Significance	n	
	Rho	(one sided)		
9 M1	$-0.737**$	0.008	10	
9 M ₂	$-0.782**$	0.004	10	
9 M3	$0.321**$	0.183	10	
9 M4	$-0.782**$	0.004	10	
9_M5	$-0.806**$	0.002	10	
9 M ₆	$-0.648**$	0.021	10	
Pooled Data	$-0.509**$	0.000	60	
** significant at a 1% level; * significant at a 5% level				

