Austrian Stochastic Days 2024

University of Innsbruck
September 4–6, 2024

Wednesday, September 4

13:30 – 14:00	*Registration*
14:00 – 14:50	**Keynote Talk: Nina Gantert (HSB 1)**
	Biased random walk on dynamical percolation
14:50 – 15:20	*Coffee Break*
15:20 – 18:00	**Parallel Sessions**
	Mathematical Finance (HSB 1)
	Statistical Mechanics (HSB 2)

Thursday, September 5

09:00 – 09:50	**Keynote Talk: Fabio Toninelli (HSB 1)**
	Mixing time of random tilings
09:50 – 10:20	*Coffee Break*
10:20 – 13:00	**Parallel Sessions**
	Stochastic PDE/ODE (HSB 1)
	Random Walks (HSB 2)
13:00 – 13:30	*Lunch (Sandwiches provided)*
13:30 – 16:30	*Hike to Rauschbrunnen*
18:00	*Conference Dinner at Planötzenhof*

Friday, September 6

09:00 – 09:50	**Keynote Talk: Jan Maas (HSB 1)**
	Anisotropic transport inequalities and the infinitesimal model
09:50 – 10:20	*Coffee Break*
10:20 – 12:35	**Parallel Sessions**
	Many facets of Stochastics (HSB 1)
	Probabilistic Combinatorics (HSB 2)

Keynote talks

Nina Gantert (Munich University of Technology)

Biased random walk on dynamical percolation

As an example for a random walk in random environment, we study biased random walk for dynamical percolation on the d-dimensional lattice. We establish a law of large numbers and an invariance principle for this random walk using regeneration times. Moreover, we verify that the Einstein relation holds, and we investigate the speed of the walk as a function of the bias. While for d = 1 the speed is increasing, we show that in general this fails in dimension $d \geq 2$. As our main result, we establish two regimes of parameters, separated by a critical curve, such that the speed is either eventually strictly increasing or eventually strictly decreasing. This is in sharp contrast to the biased random walk on a static supercritical percolation cluster, where the speed is known to be eventually zero. Based on joint work with Sebastian Andres, Dominik Schmid and Perla Sousi.

Fabio Toninelli (Vienna University of Technology)

Mixing time of random tilings

I will discuss rhombus tilings of a (tilable) finite subset D of the plane and Markov chains (Glauber dynamics) that are reversible with respect to the uniform measure over all possible tilings. What is the mixing time T_{mix}? Under some natural conditions on the domain D, it is expected that T_{mix} grows like $L^{2+o(1)}$ in continuous time, where L is the diameter of D. I will discuss recent, less recent and ongoing progress in this direction, as well as some intriguing open problems (based on joint works with Benoit Laslier and ongoing work with Amol Aggarwal).

Jan Maas (IST Austria)

Anisotropic transport inequalities and the infinitesimal model

We prove upper bounds on the L^∞-Wasserstein distance between strongly log-concave probability densities and log-Lipschitz perturbations. In the simplest setting, such a bound amounts to a transport-information inequality involving the L^∞-Wasserstein metric and the relative L^∞-Fisher information. We show that this inequality can be sharpened significantly in situations where the involved densities are anisotropic. Our proof is based on probabilistic techniques using Langevin dynamics. As an application of these results, we generalise a recent result by Calvez, Poyato, and Santambrogio on the rate of convergence in Fisher's infinitesimal model from dimension 1 to arbitrary dimensions. This is joint work with Ksenia Khudiakova (ISTA) and Francesco Pedrotti (ISTA).

Invited Sessions Talks

Mathias Beiglböck (University of Vienna)

Adapted Wasserstein distance and applications in finance

In contrast to probability measures, the flow of information plays a crucial role for stochastic processes. Adapted Wasserstein distance is a variant of the classical Wasserstein distance which accounts for this difference. It turns the set of stochastic processes in a geodesic space and is strong enough to guarantee continuity of basic probabilistic operations such as optimal stopping and the Doob decomposition. In this talk we give a gentle introduction and explain applications to weak transport and mathematical finance.

Marcin Lis (Vienna University of Technology)

On Pfaffians in spin models

It is a classical result of Groeneveld, Boel and Kasteleyn that boundary spin correlations functions in Ising models on planar graphs satisfy Pfaffian relations. Here we consider the reverse question, and show that any classical ferromagnetic spin model whose correlation functions satisfy Pfaffian relations must by (up to local simplifications of the graph) an Ising model on a planar graph. Our main tool is a new (coupled) version of the Edwards–Sokal representation of the Ising model applied to two independent copies of the spin model. Joint with Diederik van Engelenburg.

Irene Tubikanec (Johannes Kepler University Linz)

Network inference in a stochastic multi-population neural mass model via approximate Bayesian computation

The aim of this talk is to infer the connectivity structures of brain regions before and during epileptic seizure. Our contributions are fourfold. First, we propose a 6N-dimensional stochastic (ordinary) differential equation (SDE) for modelling the activity of N coupled populations of neurons in the brain. This model further develops the (single population) stochastic Jansen and Rit neural mass model, which describes human electroencephalography (EEG) rhythms, in particular signals with epileptic activity. Second, we construct a reliable and efficient numerical scheme for the SDE simulation, extending a splitting procedure proposed for one neural population. Third, we propose an adapted Sequential Monte Carlo Approximate Bayesian Computation algorithm for simulation-based inference of both the relevant real-valued model parameters as well as the 0,1-valued network parameters, the latter describing the coupling directions among the N modelled neural populations. Fourth, after illustrating and validating the proposed statistical approach on different types of simulated data, we apply it to a set of multi-channel EEG data recorded before and during an epileptic seizure. The real data experiments suggest, for example, a larger activation in each neural population and a stronger connectivity on the left brain hemisphere during seizure.

Wolfgang Woess (Graz University of Technology)

Title: Some remarks on hyperbolic branching Brownian motion

Abstract: Euclidean branching Brownian motion (BBM) has been intensively studied during many decades by renowned researchers. BBM on hyperbolic space has received less attention. A profound study of Lalley and Sellke (1997) provided insight on the recurrent, resp. transient regimes of BBM on the Poincare' disk. In particular, they determined the Hausdorff dimension of the limit set on the boundary circle in dependance on the fission rate of the branching particles. In the present talk, some further features are exhibited, such as the rate of the maximal hyperbolic distance to the starting point and the behaviour of the empiricial distributions of the branching population, as time goes to infinity.

Stephan Wagner (Graz University of Technology)

Fringe subtrees and additive functionals: recurring themes in the study of random trees

Results on the distribution of tree parameters play a key role in the analysis of algorithms, phylogenetics and other areas, and it is therefore desirable to develop general methods for this purpose. I will discuss how the notion of additive tree functionals is a unifying tool to obtain limit theorems for a wide variety of tree parameters under different models of randomness. These results will be illustrated with topical examples and applications.

Contributed session “Mathematical Finance”

Stefan Schrott (University of Vienna)

Dual Attainment for Weak Optimal Transport

Weak optimal transport is a generalization of optimal transport that allows for costs that cover many optimization problems outside the realm of classic optimal transport, while still permitting the same results concerning primal existence and weak duality as in the classical case. However, the question of dual attainment has remained open so far. Our main contribution is to establish the existence of dual optimizers, thus extending the fundamental theorem of optimal transport to the weak transport case. This is based on joint work with Mathias Beiglböck, Gudmund Pammer, and Lorenz Riess.

Evgeny Kolosov (ETH Zurich)

On arbitrage-free prices of American Options

One of the key questions in robust finance is the study of arbitrage-free models consistent with the observed prices of various derivatives. Most of the literature focuses on the case where the observations are the prices of a certain set of European options. It is known that the knowledge of the full curve of European option prices for given maturities allows to determine the marginal distributions of the underlying asset price for those maturities, and the condition for the existence of a compatible arbitrage-free model is equivalent to these distributions being in convex order. However, if the observations are prices of American options, the problem becomes significantly more complex, as their prices do not provide complete information regarding the marginal distributions. In this work, we investigate the conditions for the existence of arbitrage-free models consistent with the observed prices of American options. This leads to an extension of the concept of convex order, which we term “biased convex order.” We also study the properties of this order and provide a Strassen-type result characterizing it.

Marcus Wunsch (ZHAW School of Management and Law)

Drawdown duration and portfolio optimization

Drawdowns of stochastic processes have two key dimensions: depth (the relative shortfall versus the running maximum) and duration (the time elapsed since the last maximum). A considerable amount of research has been dedicated to drawdown depths. However, the investigation of drawdown durations has been comparatively limited, despite their significant practical importance: a portfolio wealth process suffering from prolonged drawdowns not only diminishes investors’ funds, but also threatens the asset manager’s business due to the risk of outflows. In this talk, I will review results related to the distributional properties of drawdown durations for diffusion processes [1] and their connections with Parisian options [2], quantile options [3], and drawdown options [4]. Furthermore, I will introduce and discuss several open stochastic optimization problems based on the duration of drawdowns that are motivated by practical considerations, and present preliminary findings utilizing neural networks. This research is based on joint work with Songyan Hou (ETH Zürich).

Atsushi Takeuchi (Tokyo Woman’s Christian University)

Wasserstein distance on solutions to jump-type stochastic differential equations

Consider jump processes X and Y determined by stochastic differential equations with jumps. In this talk, we shall focus on the estimates of the Wasserstein distance $d_W(X_t, Y_t)$. As one of the applications, the comment on the subordinated Brownian motion on a Riemannian manifold will be given.

Benedict Bauer (University of Vienna)

Martingale Bridges with prescribed domain

In discrete time financial markets, a sufficient condition for the absence of arbitrage is the existence of a martingale pricing measure that satisfies market-imposed constraints. From a model-free perspective, this leads to investigating the existence of martingale bridges with prescribed domains. However, conventional convex ordering requirements on marginal distributions are insufficient to guarantee their existence. To address this, we reformulate Strassen's theorem, moving away from the convex envelope approach to one that aligns with the geometry of the prescribed domain. This new formulation gives rise to duality results applicable to robust utility maximization with fixed marginal constraints. This is joint work with Christa Cuchiero.

Contributed session “Statistical Mechanics”

Markus Heydenreich (University of Augsburg)

Critical exponents for dependent percolation
We investigate spatial random graph models whose vertices are given as Poisson processes on Euclidean resp. hyperbolic space. Edges are inserted between any pair of points independently with probability depending on the distance of the two endpoints and, possibly, independent vertex marks. Upon variation of the Poisson density, a percolation phase transition occurs under mild conditions: for low density there are finite connected components only, while for large density there is an infinite component almost surely. Our interest is on the transition between the low- and high-density phase, where the system is critical. We establish the existence of various mean-field critical exponents that characterise the phase transition, in high-dimensional Euclidean space as well as in hyperbolic space. Based on joint work with Matthew Dickson.

Andreas Klippel (TU Darmstadt)

Comparison of the random loop and random path model to percolation
In recent years, many models in mathematical physics have been encoded into graphical models, which are more accessible through the lens of probability theory. These graphical models often exhibit a natural percolation structure. One such model is the Random Loop Model introduced by Daniel Ueltschi. Peter Mühlbacher showed that the loop threshold for the Random Loop Model with $\theta = 1$ is larger than the percolation threshold due to a so-called blocking events. Using a different approach, we improve the bound for the inverse temperature he obtained and provide an explicit formula for blocking events. This was a joint work with Volker Betz and Mino Kraft.

In another work, we compare the Random Path Model, introduced by Benjamin Lees and Lorenzo Taggi, to its induced dependent percolation. We show that the loop threshold of the Random Path Model is larger than the threshold of the dependent percolation it induces. Employing the concept of blocking events, we show that these events occur with positive density by using a mapping from non-blocking to blocking. This research is conducted in collaboration with Benjamin Lees. In my talk, I will provide an overview of the two different models and highlight where the ideas in the proofs overlap. If time permits, I will also explain the major proof ideas.

Ritvik Radhakrishnan (ETH Zurich)

Quantitative correlation inequalities in two dimensional critical percolation
We prove a general interpolation formula which, in particular, gives a quantitative and unified proof of the FKG and BK inequalities. We use this to show that in critical bond percolation on the square lattice the two arm exponent is strictly larger than the one arm exponent. This answers a question of Schramm and Steif (2010), and shows that their proof of the existence of exceptional times on the triangular lattice also applies to the square lattice. This method also gives a new proof of a result due to Beffara and Nolin (2011) stating that monochromatic arm exponents are strictly larger than polychromatic arm exponents.

Larissa Richards (University of Leeds)

On random planar growth
I will give a talk on two models of random growing clusters on the complex plane: 1. SLE which is a random growth process based on Lowner evolution with driving parameter a one-dimensional Brownian motion and 2. ALE which is built by iterated composition of random conformal maps. It can still be described using Lowner evolution but with a random measure as the driving parameter. An interesting feature of these models is that they can be used to define natural off-lattice analogues of several fundamental discrete models, such as Eden growth, Dielectric Breakdown and Diffusion Limited Aggregation, by tuning the correlation between the defining maps appropriately. In this talk, I will introduce both models and then discuss a rate of convergence for certain discrete lattice models to SLE and then discuss some of the large-scale properties such as scaling limits and fluctuation of ALE.

Moritz Dober (University of Vienna)

Antiferromagnetic regimes in the Ashkin–Teller model

The Ashkin–Teller (AT) model can be represented by a pair of Ising spin configurations with coupling constants J and J' for each, and U for their product. We discuss this representation on the integer lattice $\mathbb{Z}^d$ for $d \geq 2$ when negative coupling constants are present. Using the bipartite structure of $\mathbb{Z}^d$, we argue that it suffices to study the regime $J, J' > 0 > U$. We present results which confirm both ferromagnetic and antiferromagnetic behaviour in different parts of this regime. In the planar case $d = 2$, the AT model is related to the six-vertex model. Time permitting, we also discuss the corresponding results for the latter.

Contributed session "Stochastic PDE/ODE"

Johannes Rimmele (University of Augsburg)

Stabilization by rough noise for an epitaxial growth model

In this article we study a model from epitaxial thin-film growth. It was originally introduced as a phenomenological model of growth in the presence of a Schwoebbel barrier, where diffusing particles on a terrace are not allowed to jump down at the boundary. Nevertheless, we show that the presence of arbitrarily small space-time white noise due to fluctuations in the incoming particles surprisingly eliminates all non- linear interactions in the model and thus has the potential to stabilize the dynamics and suppress the growth of hills in these models.

Matija Vidmar (University of Ljubljana)

Extending the noise of splitting to its completion

The noise of splitting introduced by Warren [arXiv:math/9911115] admits an extensions to a largest class of domains indexing its completion (which is a concept due to Tsirelson [Ann. Probab. 42(1):311-353]). The domains to which this extension is possible are characterized by a certain stability of the times of (local) maxima of a Brownian motion to perturbation. Some domains are totally unstable with respect to such perturbation and to them the extension cannot be made. (Joint work with J. Warren.)

Benjamin Robinson (University of Klagenfurt)

Bicausal optimal transport for SDEs with irregular coefficients

Many natural phenomena that exhibit randomness can be modelled by stochastic differential equations (SDEs), often having less regularity than the classical case of SDEs with Lipschitz coefficients. In such settings, we are interested in quantifying model uncertainty and the impact of model choice on the value of stochastic optimisation problems. To this end, we seek an appropriate notion of distance on the space of models. In particular, we study the adapted Wasserstein distance between the laws of SDEs. This is a special case of a bicausal optimal transport problem, in which the classical optimal transport problem is constrained to respect the flow of information inherent in stochastic processes.

Under minimal regularity assumptions on the coefficients, we show that the value of the bicausal optimal transport problem between the laws of one-dimensional SDEs is attained by the synchronous coupling. This is the coupling induced by taking a common Brownian motion as the driving noise for each SDE. Our proof is based on a discretisation method, exploiting monotonicity properties of the resulting discrete-time processes. A key tool in our work is a transformation-based semi-implicit Euler—Maruyama scheme for SDEs whose drift coefficient may have discontinuities and exponential growth. We prove the first strong existence and uniqueness result for such SDEs, and we obtain strong convergence rates for the implicit scheme. Moreover, our results provide a method for efficient computation of the adapted Wasserstein distance. This is joint work with Michaela Szölgyenyi (University of Klagenfurt)

Verena Schwarz (University of Klagenfurt)

Higher-order approximation of jump-diffusion McKean–Vlasov SDEs

In this talk we study the numerical approximation of the jump-diffusion McKean-Vlasov SDEs with super-linearly growing drift, diffusion and jump-coefficient. In a first step we use derive the corresponding interacting particle system and define for this a general Milstein-type approximation. Making use of the propagation of chaos result and investigating the error of the Milstein-type scheme we provide convergence results for the scheme. In a second step we discuss potential simplifications of the numerical approximation scheme for the direct approximation of the jump-diffusion McKean-Vlasov SDE. Lastly, we present the results of our numerical simulations. This is joint work with Sani Biswas, Chaman Kumar and Christoph Reisinger.

Boris Jidjou Moghomye (Montanuniverstät Leoben)

On a stochastic Chemotaxis-fluid model

In this talk, we present an existence result for a stochastic model describing the dynamic of collective behaviour of oxygen-driven swimming bacteria in an aquatic fluid flowing in a bounded domain influenced by random external forces. The considered model consists of the stochastic Navier-Stokes equations coupled with Keller–Segel equations.

Contributed session "Random Walks"

Timo Vilkas (Lund University)

About left-continuous random walk on $\mathbb{Z}$ and the parity of its hitting times
Arguably the simplest generalization of simple random walk on Z is to limit just the down steps to size one: so-called left-continuous walks. In this talk we determine their parity based hitting probabilities to be able to draw conclusions for random walk that is not quite left-continuous.

Lorenz Gilch (University of Passau)

Capacity of the Range of Random Walks on Free Products of Graphs
In this talk we study the asymptotic capacity of the range of random walks. First, I will give a quick introduction to the concept of the capacity of the range, which has been investigated mostly on $\mathbb{Z}^d$. However, there are not many results going beyond. We will focus in this talk on random walks on free products of graphs, where we sketch the basich idea of the proof that the asymptotic capacity of the range is almost surely constant and strictly positive. Further results like a central limit theorem and analyticity of the capacity will be presented.

Aleksandr Tarasov (University of Bielefeld)

Asymptotic expansions of random walks conditioned to stay positive
Consider a one-dimensional random walk S_n with i.i.d. increments with zero mean and finite variance. We study the asymptotic expansion for the tail distribution $\mathbb{P}(\tau_x > n)$ of the first passage times $\tau_x := \inf\{n \geq 1 : x + S_n \leq 0\}$ under minimal moment conditions. We also derive an asymptotic expansion for local probabilities $\mathbb{P}(S_n = x, \tau_0 > n)$ in two different regimes: for $x = o(\sqrt{n})$ and for $x \sim \sqrt{n}$.

Attila Lovas (Alfréd Rényi Institute of Mathematics, Budapest)

On the strong stability of ergodic iterations
We revisit processes generated by iterated random functions driven by a stationary and ergodic sequence. Such a process is called strongly stable if a random initialization exists, for which the process is stationary and ergodic, and for any other initialization, the difference between the two processes converges to zero almost surely. Under some mild conditions on the corresponding recursive map, without any condition on the driving sequence, we show the strong stability of iterations. Several applications are surveyed such as generalized autoregression and queuing. Furthermore, new results are deduced for Langevin-type iterations with dependent noise and for multitype branching processes.

Nikita Elizarov (University of Bielefeld)

Coexisting of branching populations
Consider two one-dimensional branching populations (Z_n^1, Z_n^2) in a joint random environment. Quenched distributions of Z_n^1 and Z_n^2 are assumed independent. Thus, the dependence between populations is caused by the environment only. We are interested in the asymptotic behaviour of coexisting probability $\mathbb{P}(Z_n^2 > 0, Z_n^2 > 0)$. We are going to show that this problem is deeply connected to a two-dimensional random walk $\hat{S}_n$ conditioned to stay in a cone. $\hat{S}_n$ is the Doob h-transform of a random walk S_n having i.i.d. increments with zero mean and finite variance and killed at leaving the cone. For the process $\hat{S}_n$ we estimate the probability of coming close to the boundary of the cone. This will give us upper and lower bounds for the coexistence probability.

Contributed session “Many facets of Stochastics”

Daniel Bartl (University of Vienna)

A high dimensional Dvoretzky–Kiefer–Wolfowitz inequality

The DKW inequality is a non-asymptotic, high probability estimate on the L_∞ distance between the distribution function of a real-valued random variable and its empirical counterpart. Little was known on generalisations of that inequality to high dimensions, where instead of a single random variable one is interested in the behaviour of empirical distribution functions uniformly over a family of random variables. We show that the behaviour of various notions of distances (including the L_∞ one) between the empirical and actual distributions in the given family can be fully characterised in terms of some (rather surprising) intrinsic complexity parameter of the family. Based on joint work with Shahar Mendelson.

Johannes Wiesel (Carnegie Mellon University)

Bounding adapted Wasserstein metrics

The Wasserstein distance $\mathcal{W}_p$ is an important instance of an optimal transport cost. Its numerous mathematical properties as well as applications to various fields such as mathematical finance and statistics have been well studied in recent years. The adapted Wasserstein distance $\mathcal{AW}_p$ extends this theory to laws of discrete time stochastic processes in their natural filtrations, making it particularly well suited for analyzing time-dependent stochastic optimization problems. While the topological differences between $\mathcal{AW}_p$ and $\mathcal{W}_p$ are well understood, their differences as metrics remain largely unexplored beyond the trivial bound $\mathcal{W}_p \lesssim \mathcal{AW}_p$. This paper closes this gap by providing upper bounds of $\mathcal{AW}_p$ in terms of $\mathcal{W}_p$ through investigation of the smooth adapted Wasserstein distance. Our upper bounds are explicit and are given by a sum of $\mathcal{W}_p$, Eder's modulus of continuity and a term characterizing the tail behavior of measures. As a consequence, upper bounds on $\mathcal{W}_p$ automatically hold for $\mathcal{AW}_p$ under mild regularity assumptions on the measures considered. A particular instance of our findings is the inequality $\mathcal{AW}_1 \leq \sqrt{\mathcal{W}_1}$ on the set of measures that have Lipschitz kernels.

Our work also reveals how smoothing of measures affects the adapted weak topology. In fact, we find that the topology induced by the smooth adapted Wasserstein distance exhibits a non-trivial interpolation property, which we characterize explicitly: it lies in between the adapted weak topology and the weak topology, and the inclusion is governed by the decay of the smoothing parameter. This talk is based on joint work with Jose Blanchet, Martin Larsson and Jonghwa Park.

Alexander Steinicke (Montaniversität Leoben)

A quantification of almost sure convergence with applications to Brownian paths and discrete martingales

We first present an elementary, but useful, quantitative generalization of the first Borel–Cantelli lemma. The idea is to translate good rates of convergence of probability events into higher order moments of the 'overlap statistics'. That is, it can quantify almost sure convergence of a random variable sequence $(X_n)_{n\geq 1}$ to a random variable X in terms of the number of occurrences of the error events $\{|X_n - X| > \epsilon\}$ and the last index where such an error event happens. We provide various applications, e.g. to martingale convergence, to properties of Brownian paths and to branching processes. This is joint work with Michael Högele and Luisa Estrada.

Panagiotis Spanos (Ruhr-Universität Bochum)

Intersection probabilities of subspaces of constant curvature
In this talk, we will present established results regarding the intersection of a random linear subspace with an affine subspace in Euclidean space. We will then extend this model to spaces of constant negative curvature. In this new setting, interesting phenomena arise, such as the probability that the intersection of two such spaces can be empty with strictly positive probability. We will discuss the asymptotic properties as the dimensions of the spaces increase to infinity and present that a phase transition occurs if the curvature of the space depends on the dimension. This talk is based on joint work with E. Sönmez and C. Thäle.

David Padilla-Garza (Hebrew University)

Emergence of a Poisson process in weakly interacting particle systems
This talk will be about an interacting particle system driven by two forces: a repulsive pairwise interaction between them, and a confining potential. At a positive temperature, their behavior is driven by the Gibbs measure associated with this Hamiltoninan.In this talk, we will be interested in a certain class of "weakly interacting " pairwise interactions. For this kind of interaction, we will show that the local behavior of the system is asymptotically given by a Poisson Point Process. We show this under more general assumptions on the temperature scaling than previous works in the literature.

Contributed session "Probabilistic Combinatorics"

Ivailo Hartarsky (TU Wien)

Catalan Percolation

In Catalan percolation, one declares the edges $\{i, i+1\}$ for $i \in \mathbb{Z}$ *occupied* and each edge $\{i, j\} \subset \mathbb{Z}$ with $j \geq i+2$ *open* independently with probability p. For $k \geq i+2$, we recursively define $\{i, k\}$ to be *occupied*, if $\{i, k\}$ is open and both $\{i, j\}$ and $\{j, k\}$ are occupied for some $j \in \{i+1, \ldots, k-1\}$. The model was introduced by Gravner and Kolesnik in the context of polluted bootstrap percolation, but is tightly linked with Catalan structures and oriented percolation. We establish that the critical parameter of the model is strictly between the natural lower and upper bounds given by 1/4 and the critical probability of oriented site percolation on $\mathbb{Z}^2$ respectively. The most challenging part of the proof is a strict inequality for the critical parameter of an oriented percolation model with non-decaying infinite range dependencies, not relying on the Aizenman–Grimmett argument for essential enhancements. The talk is based on joint work with Eleanor Archer, Brett Kolesnik, Sam Olesker-Taylor, Bruno Schapira and Daniel Valesin available at https://arxiv.org/abs/2404.19583.

Michael Anastos (IST Austria)

Partitioning problems via random processes

There are a number of well-known problems and conjectures about partitioning graphs to satisfy local constraints. For example, the majority colouring conjecture of Kreutzer, Oum, Seymour, van der Zypen and Wood states that every directed graph has a majority 3-colouring i.e. a colouring of its vertices with 3 colours such that for every vertex v, at most half of the out-neighbours of v have the same colour as v. We prove that this conjecture holds for almost all digraphs of any given density: For every $p = p(n) \in [0, 1]$, the binomial random digraph $D(n, p)$ has a majority 3-colouring with high probability. Our proof relies on a carefully designed randomised colouring procedure that iteratively converges to a majority 3-colouring.

Nicolas Allen Smoot (University of Vienna)

Identifying Integer Partition Congruences

A major subject in additive number theory is the study of integer partitions. For example, let $p(n)$ count the number of partitions of n; so $p(4) = 5$, since 4 has 5 different partitions: $4, 3+1, 2+2, 2+1+1, 1+1+1+1$. At first sight the arithmetic properties of $p(n)$ appear random. For all n up to, say, 10000, nearly half of the values for $p(n)$ are even, and nearly 1/3 of the values are divisible by 3. On the other hand, about 36% of such values are divisible by 5—noticeably more than the expected 20%. This peculiarity signifies the remarkable fact that $p(5n+4)$ is always divisible by 5. Similar divisibility properties apply to a variety of different functions in the theory of partitions. In this talk we will show how similar statistical approaches can be used to find divisibility properties for other partition functions, and discuss the impact that these properties have had on modern number theory.

Elia Bisi (University of Florence)

Random matrices, Young diagrams, and trees

We consider λ-shaped random matrices, whose entries are i.i.d. in the boxes of a given Young diagram λ and zero elsewhere. In particular, we study their limiting spectral distribution when the matrix size gets large. We express the moments of such a distribution in terms of a combinatorial object which we call λ-plane trees: these are trees whose vertices are labelled in a way that is 'compatible' with λ. Based on joint work with Fabio D. Cunden.