

Department of Mathematics



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CUR for Low-Rank Approximation of Nonlinear Matrix Equations and Extension to Tensor Differential Equations

Matrix differential equations (MDEs) and high-dimensional tensor differential equations (TDEs) are omnipresent in science and engineering. Example applications include the Schrödinger equation, probability density function transport equations in turbulent combustion, the Fokker-Planck equation, the Boltzmann transport equation, and the Hamilton-Jacobi-Bellman equation, among others. However, performing computational tasks involving high-dimensional problems, or even storing them, suffers from the curse of dimensionality: the total degrees of freedom increase exponentially as the dimension grows. Various low-rank approximations have been developed to mitigate this issue by leveraging multi-dimensional correlations. These dimension reduction techniques aim to decrease the total number of degrees of freedom while allowing for a controllable loss of accuracy. In this seminar, we review the recent advancements in dynamical low-rank approximation (DLRA), which provides a rigorous mathematical framework for solving MDEs and TDEs on low-rank manifolds. We particularly focus on the computational cost issues of solving DLRA equations. We present a new formulation for solving nonlinear MDEs and TDEs based on CUR and cross low-rank approximation algorithms. We present CUR algorithms based on the Discrete Empirical Interpolation Method (DEIM). We show that the DEIM CUR algorithms can achieve nearoptimal accuracy for general nonlinear MDEs and TDEs. Several applications are presented, including turbulent combustion, sensitivity analysis, and uncertainty quantification.

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