Titles and abstracts

Groups, Languages, and Random Walks

Cortona, June 02-07, 2024

1. **Laurent Bartholdi**
   Title: Automatic Actions
   Abstract: I will present a general notion of automatic action, based on Büchi automata, and show how it unifies a large number of subclasses, in particular the automatic groups by Cannon, Thurston et al.; the transducer groups by Aleshin, Grigorchuk, Sushchansky, Sidki et al.; substitutional subshifts; and complex dynamics. I will present some algorithms for these groups, and in particular show under an extra condition (boundedness) that their orbit relation is computable and regular. This will have strong decidability consequences, such as that the order problem, aperiodicity, minimality, etc. for automatic transformations is decidable. I will detail applications to symbolic dynamics: in particular, it is decidable whether a substitutional subshift is aperiodic, minimal, topologically transitive, etc.; and to complex dynamics: it is decidable e.g. if a Julia set is a Sierpinski carpet. (30 min. talk)

2. **Daniela Bertacchi**
   Title: An epidemic model in inhomogeneous environment
   Abstract: We introduce an epidemic model on the complete graph $\mathbb{K}_n$ on n vertices in a non-homogeneous setting, where the vertices may have distinct types. Different types differ in the probability of getting infected, and/or in the capacity of infecting other vertices. This generalizes the model in a paper of Comets et al, 2014. We prove laws of large numbers and central limit theorems for the total duration of the process and for the number of infected vertices, respectively, when $n \to \infty$. By coupling the epidemic model with a Poisson process, we also obtain continuous-time counterparts of the above-mentioned limit results. Moreover, we also prove that when all individuals have the same spread capacity, then a population with inhomogeneous susceptibility is less affected by the epidemics than a homogeneous population. This is joint work with Jurgen Kampf, Ecaterina Sava-Huss and Fabio Zucca. (30 min. talk)

3. **Emmanuel Breuillard**
   Title: Limit theorems and equidistribution on nilpotent Lie groups.
   Abstract: Generalizing the classical limit theorems of probability theory to non-commutative Lie groups is a long standing question that has been studied since the 1950s. Nilpotent Lie groups form a rich class of groups where such theorems can be formulated. In this talk I will present joint work with Timothee Benard in which we use a Fourier analytic approach based on a non-commutative Weyl differencing technique to establish very complete results regarding the Central Limit Theorem, Berry-Essen bounds and their local counterparts for iid random walks on nilpotent Lie groups. Interesting new phenomena arise in the non-centered setting. (45 min. talk)

4. **Jeremie Brieussel**
   Title: Furstenberg entropy spectrum of stationary actions of special linear groups
   Abstract: A measure on a space is invariant if it is preserved by the action of each element of the group. A measure is stationary if it is equal to the average, according to some probability measure, of its translates by group elements. The Furstenberg entropy is an invariant of

stationary systems that measures the lack of invariance. In particular it vanishes for invariant measures. The aim of the talk is to give a description of all possible values taken by the Furstenberg entropy of stationary actions of $SL(d,\mathbb{R})$ endowed with a reasonable probability measure. It is a joint work with Tianyi Zheng. (30 min. talk)

5. **Sara Brofferio**
Title: Local stability for affine recursions in higher dimensions
Abstract: Consider a stochastic dynamical system that is a Markov process X_n^x on $\mathbb{R}^d$ obtained by applying some i.i.d. random transformations F_n to a starting point $x \in \mathbb{R}^d$:

$$X_n^x = F_n(X_{n-1}^x) = F_n \cdots F_1(x), \quad X_0^x = x.$$

A crucial question is whether such systems are sensitive to perturbations of the starting point x, in particular, whether the distance between two trajectories X_n^x and X_n^y starting from two different points $x \neq y$ converges to zero. We are particularly interested in systems that are not necessarily globally stable but exhibit *local* stability, meaning that the distance $|X_n^x - X_n^y|$ converges to zero when the processes are observed within a compact window $K \subset \mathbb{R}^d$:

$$|X_n^x - X_n^y| 1_K(X_n^x) \to 0 \text{ as } n \to +\infty.$$

While such phenomena have been highlighted for several examples of stochastic dynamical systems in dimension $d = 1$ in critical cases between contraction and dilation, they are less understood in higher dimensions. The goal of this talk is to present some results on the local stability properties for the random affine recursion X_n^x induced by the action of the random walk on the (semi)group of affine transformations of the Euclidean space $\mathbb{R}^d$, which is given by

$$X_n^x = A_n X_{n-1}^x + B_n,$$

where $(A_n, B_n) \in M(d) \times \mathbb{R}^d$ is an i.i.d. sequence in the critical case where the Lyapunov exponent of the matrices is zero. (30 min. talk)

6. **Matteo Cavaleri**
Title: Generic world problem and Banach densities in finitely generated groups
Abstract: In this talk we will survey the results obtained so far in the search for finitely generated groups with unsolvable generic Word Problem (WP). We will start with the existence (A. Myasnikov, D. Osin) of algorithmically finite groups, whose Equality Problem (EP) is extremely undecidable and it is far from being generically solvable. We will consider the specific case of amenable groups, providing examples of finitely presented groups with unsolvable generic EP. Thanks to the introduction of certain Upper Banach (UB) densities, for which the concept of generic word problem is particularly natural and invariant with respect to the choice of generators, we managed to show that solvability of generic EP implies solvability of the whole WP, for every finitely generated group. We then exploit another definition of generic EP, which turns out to be equivalent to generic WP. Finally, we characterize in different ways the class of groups with unsolvable UB-generic WP, proving that it contains that of algorithmically finite groups, and it is contained in that of groups with unsolvable generic WP. This shows that algorithmically finite groups are also examples of computably presented groups with unsolvable generic WP. (30 min. talk)

7. **Michel Coornaert**
Title: Stable finiteness of group algebras of surjunctive groups and model theory
Abstract: Using algebraic geometry methods, Xuan Kien Phung has shown that the group algebra of a surjunctive group is always stably finite. In other words, every group satisfying Gottschalk's conjecture must also satisfy Kaplansky's stable finiteness conjecture. I will explain a proof of Phung's result based on first-order model theory. This is work in collaboration with Tullio Ceccherini-Silberstein and Xuan Kien Phung. (30. min talk)

8. **Corentin Correia**
Title: Odomutants and quantitative orbit equivalence
Abstract: Two measurable bijections of a standard probability space are orbit equivalent if they have the same orbits up to conjugacy. In recent years, odometers have been a central class of systems for explicit constructions of orbit equivalences, using their combinatorial structure. In this talk we introduce a construction of orbit equivalence between odometers and new systems that we call odomutants. The starting point for this notion is a construction of Feldman in 1976, which enables us to get a first flexibility result about even Kakutani equivalence. Here we deal with a second result, about entropy. It follows from work of Kerr and Li that if the cocycles are log integrable, the entropy is preserved. Our construction of odomutants shows that their result is optimal, namely we find odomutants of positive entropy orbit equivalent to an odometer, with almost log integrable cocycles. (15 min. talk)

9. **Wojciech Cygan**
Title: Discrete Feynman–Kac evolutions with confining potentials
Abstract: We study a certain discrete-time Markov evolution in a countably infinite state space that describes the motion of a single particle which is confined through an unbounded potential. From the probabilistic point of view it is a Markov chain whose paths are killed with random intensity coming from an external potential. Its (non-conservative) transition semigroup is a counterpart of the classical Feynman–Kac semigroup. We are mainly interested in long-range Markov chains whose generators are nonlocal (in a specific sense) discrete operators. In the talk we will give a short introduction to this topic. We will first discuss sharp estimates for functions that are (sub-)harmonic in infinite sets with respect to the discrete Feynman–Kac operators and we will present some applications. These results will be compared with respective estimates for the case of the nearest-neighbour random walk which evolves on an infinite graph of finite geometry. Further, we shall discuss intrinsic ultra- and hypercontractivity properties (together with their progressive-in-time versions), and applications to uniform (quasi-)ergodicity properties on ℓ_p-spaces. Our approach is based on the *direct step property* (DSP in short) of the underlying Markov chain and it encompasses a fairly general class of processes and operators. We will present a few constructions leading to DSP Markov chains and illustrate them by various examples. This is a joint ongoing project with Kamil Kaleta (Wrocław University of Science and Technology), Rene Schilling (TU Dresden) and Mateusz Śliwiński (Wrocław University of Science and Technology). (30 min. talk)

10. **Adam Dor-On**
Title: Space-time Martin boundary and ratio-limit boundaries
Abstract: Ratio-limit boundaries were first studied for their applications to Toeplitz C^*-algebras of random walk, but are interesting in their own right for measuring new types of behavior at infinity. For the purpose of describing Toeplitz C^*-algebras of random walks, new boundaries need to be identified in more precise terms. One such boundary is the so-called space-time Martin boundary, as studied by Lalley for random walks on the free group. In this talk we will discuss ratio-limit boundaries and some work in progress on space-time Martin boundaries of random walks on discrete groups. The space-time Martin boundary is related to the notion of stability studied by Picardello and Woess, which will assist us in attaining insight and descriptions of the space-time Martin boundaries for random walks on $\mathbb{Z}^d$ and on hyperbolic groups. (30 min. talk)

11. **Anna Erschler**
Title: Følner functions, return probability and shape of Følner sets
Abstract: Based on joint works with Josh Frisch and Ivan Mitrofanov. (45 min. talk) **Online Talk**.

12. **Behrang Forghani**
Title: Harmonic measures and convex algebra
Abstract: The Poisson boundary of a random walk on a group is a probability space used to study the long-term behavior of the random walk. Because the group naturally acts on the Poisson boundary, various questions regarding the structure of this action can be studied. In this short talk, I will show that the set of probability measures with equivalent harmonic measures may not necessarily form a convex algebra (a convex algebra is closed under convex combinations and convolutions). To provide such examples, I will compute the harmonic measures for probability measures distributed on words of length at most 2 in the modular group $PSL(2,\mathbb{Z})$. This talk is based on joint work with Vadim Kaimanovich. (15 min. talk)

13. **Ilya Gekhtman**
Title: Random walks vs. balls
Abstract: Consider a positive entropy random walk on a countable group G which acts compactly by isometries on a metric space X. What is the minimal exponential growth rate in the Cayley graph of a subset of G in which a positive proportion of random walk trajectories spend a positive proportion of time? We will show it to be the entropy divided by the drift of the random walk. This is joint work with Arie Levit. (15 min. talk)

14. **Agelos Georgakopoulos**
Title: A Notion of Dimension based on Probability on Groups
Abstract: Probability on Groups studies how properties of a group G, such as amenability or growth rate, influence the outcome of random experiments, such as random walk or percolation, carried out on (a Cayley graph of) G. But can we learn something new about G by studying such experiments? I will survey some results in the area and introduce a notion of "dimension" of a group that arose from the hope to answer this question positively. (45 min. talk)

15. **Lorenz Gilch**
Title: Capacity of the Range of Random Walks on Free Products of Graphs
Abstract: In this talk we study the asymptotic capacity of the range of random walks. While the capacity of the range for random walks has been investigated mostly on $\mathbb{Z}^d$, there are not many results going beyond. We focus in this talk on random walks on free products of graphs, where we give a sketch of the proof that the asymptotic capacity of the range is almost surely constant and strictly positive. Further results like a central limit theorem and analyicity of the capacity will be presented. (30. min talk)

16. **Rostislav Grigorchuk**
Title: Old and new on cogrowth
Abstract: In the first 10 minutes I will recall some results from the 80th on cogrowth, including Woess' contribution. I will mention a result on the rationality of the congrowth series for normal subgroups of a free group that can be deduced from Wolfgang's work and formulate a related Conjecture. Then I will quickly describe new and relatively new developments related to cogrowth, including multivariate cogrowth and ergodicity of the boundary action. This part will be based on joint results of the speaker with P. de la Harpe, T. Ceccherini-Silberstein, V. Kaimanovich, T. Nagnibeda, J-F. Quint, A. Shaikh, and a recent observation by L.Bartholdi. (30 min. talk)

17. **Yair Hartman**
Title: Random walks and dense subgroups
Abstract: Can one relate random walks on a group with random walks on a dense subgroup of it? We develop a technique to do it in some cases. This allows us to exhibit some new and interesting phenomena in the Furstenberg-Poisson boundary theory. Joint work with Michael Björklund and Hanna Oppelmayer. (30 min. talk)

18. **Vadim Kaimanovich**
Title: Liouville property and Poisson boundary of random walks with infinite entropy: what's amiss?
Abstract: We discuss the qualitatively new properties of random walks on groups that arise in the situation when the entropy of the step distribution is infinite. (45 min. talk)

19. **Robin Kaiser**
Title: The Abelian Sandpile Model and Stabilization in Infinite Volumes
Abstract: The Abelian Sandpile model is a Markov chain that evolves in time via randomly placing particles on a given finite graph and toppling all piles of particles that get too large. For infinite graphs with almost surely one-ended uniform spanning trees it was shown that the stationary measures of the Abelian Sandpile Markov chain on a finite exhaustion of the graph converge weakly to a measure on the infinite graph, which is called the infinite volume limit of the Abelian Sandpile Model. In my talk I will focus on the question for which infinite graphs a sandpile sampled from the infinite volume measure plus one additional particle stabilizes in finitely many steps almost surely. Based on joint works with Nico Heizmann (Chemnitz) and Ecaterina Sava-Huss (Innsbruck). (15 min. talk)

20. **Konrad Kolesko**
Title: Limit theorems for branching processes
Abstract: Branching processes are a significant class of stochastic processes with numerous practical and theoretical applications. In this talk, I will present recent limit theorems concerning these processes, which enable the determination of asymptotic expansions up to Gaussian fluctuations. This work is based on recent collaborations with A. Iksanov, M. Meiners, and E. Sava-Huss. (30 min. talk)

21. **Francois Ledrappier**
Title: Dimension of the stationary measures for random walks on matrices.
Abstract: We consider the action of a countable group of matrices on the flag spaces. We discuss the dimensional properties of the stationary measures and of the invariant sets. This is a joint work with Pablo Lessa (Montevideo). (30 min. talk)

22. **Franz Lehner**
Title: Free Integral Calculus
Abstract: Free product groups provide a popular playground for the study of random walks; in particular, nearest neighbour random walks can be explicitly computed. In joint work with Kamil Szpojankowski we combine concepts from noncommutative probability with linearizations of noncommutative rational functions to provide a systematic method for the computation of Green functions of random walks with finite support. (30 min. talk)

23. **Tatiana Nagnibeda**
Title: Benjamini-Schramm and spectral convergence of Rauzy graphs
Abstract: Gérard Rauzy suggested to study subshifts over a finite alphabet with the help of graphs that form an infinite sequence of finite approximations to the original dynamical system. In this talk we will be interested in convergence properties of these sequences of graphs and of their spectra. (45 min. talk)

24. **Noema Nicolussi**
Title: Self-adjointness of infinite quantum graphs and graph ends
Abstract: In the last decades, Laplacians on metric graphs (a.k.a. quantum graphs) have become popular objects of study. The analysis of spectral properties relies on the self-adjointness of the Laplacian. Whereas on finite metric graphs the Laplacian is always self-adjoint, much less is known about the self-adjointness problem for graphs having infinitely many edges and vertices. Intuitively the question is closely related to finding appropriate

boundary notions for infinite graphs. In this talk we study the connection between self-adjointness and the notion of graph ends, a classical graph boundary introduced independently by Freuden- thal and Halin. Based on joint work with Aleksey Kostenko (Ljubljana & Vienna) and Delio Mugnolo (Hagen). (15 min. talk).

25. **Hanna Oppelmayer**
Title: Invariant Random Sub-Algebras
In a joint work with Tattwamasi Amrutam and Yair Hartman we introduce the notion of *invariant random sub-von Neumann algebra (IRA)*. This is an invariant probability measure on the space of all sub-von Neumann algebras of the group von Neumann algebra $L\Gamma$, where Γ denotes a countable discrete group. This generalizes the well-studied concept of IRSs (invariant random subgroups). An application concerning amenability is given. No pre-knowledge on von Neumann algebras is assumed. (15 min. talk)

26. **Spanos Panagiotis**
Title: Spread-out percolation on transitive graphs of polynomial growth
Abstract: We present the model of spread-out percolation on vertex-transitive graphs and a new result on graphs of superlinear polynomial growth. Specifically, this construction provides a sequence of supercritical percolation models with an expected degree converging to 1, which are well compatible with the geometry of the graph. This work extends the results of Penrose and Bollobás-Janson-Riordan, who considered the case of $\mathbb{Z}^d$ for $d \geq 2$. This is based on joint work with Matthew Tointon. (15 min. talk)

27. **Marc Peigne**
Title: A local limit theorem for oscillating random walks on $\mathbb{Z}$
Abstract: In this talk, we present some recent results on the recurrence properties of the *oscillating random walk* on $\mathbb{Z}$ and the asymptotic behavior of their return probabilities. The main tool is a renewal theorem for aperiodic sequences of operators (due to S. Gouezel) and some variations of this theorem. (30 min. talk)

28. **Kilian Raschel**
Title: Persistence for a class of order-one autoregressive processes and Mallows-Riordan polynomials
Abstract: We establish exact formulae for the (positivity) persistence probabilities of an autoregressive sequence with uniform innovations in terms of certain families of polynomials, most notably a family introduced by Mallows and Riordan as enumerators of finite labeled trees when ordered by inversions. The connection of these polynomials with the volume of certain polytopes is also discussed. Two further results provide factorizations of general autoregressive models, one for negative drifts with continuous innovations, and one for positive drifts with continuous and symmetric innovations. The second factorization extends a classical universal formula of Sparre Andersen for symmetric random walks. Our results also lead to explicit asymptotic estimates for the persistence probabilities. This is a joint work with Gerold Alsmeyer, Alin Bostan and Thomas Simon (Adv. Appl. Math., 2023). (45 min. talk)

29. **Emanuele Rodaro**
Title: On Context-Free Inverse Graphs and Their Associated Groups
Abstract: We explore recent developments in the study of context-free graphs, focusing on various generalizations of the Muller-Schupp theorem for context-free inverse graphs. Our investigation includes a weaker proof of T. Brough's conjecture on poly-context-free groups. Additionally, we extend Muller and Schupp's characterization of context-free groups by establishing equivalences between properties of quasi-transitive inverse graphs and group structures. This extension includes a group theoretic analog to the Chomsky-Schützenberger representation theorem, crucial for addressing Brough's conjecture. If time permits, we will also investigate the derivation of a specific class of co-context-free groups from these graphs and explore related conjectures. (30 min. talk)

30. **Laurent Saloff-Coste**
Title: Some stable-like random walks on finitely generated nilpotent groups
Abstract: In this talk, I will report on join work with ZQ Chen, T. Kumagai, J. Wang, T. Zheng, and also graduate student Ruoqi Zhang, in search for a theory of stable-like random walks on nilpotent groups. Keeping in mind the important role of the notion of operator stable Levy process, what are natural examples of stable-like random walks and how to study their key properties? (45 min. talk)

31. **Klaus Schmidt**
Title: Divisibility of Integer Polynomials, Homoclinic Points, and Lacunary Independence
Abstract: Let f, p, and q be Laurent polynomials with integer coefficients in one or several variables, and suppose that f divides $p + q$. We establish sufficient conditions to guarantee that f individually divides p and q. These conditions involve a bound on coefficients, a separation between the supports of p and q, and, surprisingly, a requirement on the complex variety of f called atorality.

Our proof involves a related dynamical system and the notion of summable homoclinic points of that system. We use this to establish exponential recurrence of the system, and conclude with some extensions and open problems. This is joint work with Doug Lind. (30 min. talk)

32. **Romain Tessera**
Title: IME classification of nilpotent groups
Abstract: A famous open problem in geometric group theory is the classification up to QI of nilpotent groups. It is generally conjectured that two simply connected nilpotent Lie groups are QI if and only if they are isomorphic. By an observation of Shalom, two amenable groups are QI if and only if they are uniformly measure equivalent, which in turn implies that they are integrable measure equivalent (IME). This raises the natural question of classifying nilpotent groups up to IME. By an important result of Austin, we know that IME nilpotent groups have isomorphic Carnot-graded associated Lie groups. In a joint work with Claudio Llosa Isenrich, we prove the converse. More precisely, we show that two nilpotent (connected Lie or finitely generated) groups which have isomorphic associated Carnot-graded Lie groups are integrable orbit equivalent, and so in particular are IME. (30 min. talk)

33. **Giulio Tiozzo**
The Poisson boundary of hyperbolic groups without moment conditions
Abstract: The Poisson boundary is a measure-theoretic object attached to a group equipped with a probability measure, and is closely related to the notion of harmonic function on the group. In many cases, the group is also endowed with a topological boundary arising from its geometric structure, and a recurring research theme is to identify the Poisson boundary with the topological boundary. In this talk, we prove that the Poisson boundary of a random walk with finite entropy on a non-elementary hyperbolic group can be identified with its hyperbolic boundary, without assuming any moment condition on the measure. In this generality, this identification result is new even for free groups. We will also discuss extensions of this result to other groups with hyperbolic properties, in particular acylindrically hyperbolic groups. (30 min. talk)

34. **Matthew Tointon**
Title: The structure of vertex-transitive graphs of polynomial growth
Abstract: Trofimov showed that an arbitrary vertex-transitive graph of polynomial growth is quasi-isometric to a Cayley graph. This in turn has applications to classifying those vertex-transitive graphs on which the random walk is recurrent, or on which Bernoulli bond percolation exhibits a phase transition. Some time later, Wolfgang gave a beautifully transparent proof of Trofimov's result using the theory of locally compact groups. I will describe how to modify Wolfgang's argument using recent advances in approximate group theory to obtain a finitary refinement of Trofimov's theorem. This in turn leads to quantitative finitary

refinements of the aforementioned probabilistic applications. My talk will encompass joint work with Tom Hutchcroft and with Romain Tessera. (30 min. talk)

35. **Vitali Wachtel**
Title: Random walks with square-root boundaries
Abstract: In the talk we shall consider a discrete time random walk $S(n)$ killed at the first crossing of the curve $g(t) \sim c\sqrt{t}$ with some $c \neq 0$. For the parametric family of boundaries $g(t) = c\sqrt{t+b} - a$, $b > 0$ and $a > c\sqrt{b}$ we construct a positive space-time harmonic function and determine its asymptotic properties. This function is then used to determine tail asymptotics for the time of the first crossing of $g(t)$. (45 min. talk)

36. **Ariel Yadin**
Title: Detection of homomorphisms via metric-functionals
Abstract: Metric-functionals play an analogous role to linear functionals for general metric spaces, where there is no linear structure. On Cayley graphs these objects identify with "horofunctions". As observed 20 years ago by Anders Karlsson these objects are useful also for detection or construction of homomorphisms into abelian groups. In many cases, finding such homomorphisms from finite index subgroups enables a decomposition of the group into "smaller" structures, so that one can inductively understand the geometric structure. (This was done for example by Gromov in his famous theorem regarding polynomial growth, but also in many other cases.). We can prove that it is not always possible to detect homomorphisms using the metric-functional boundary of a Cayley graph. We thus extend the notion of Cayley graphs to a broader class of metric spaces which we call "Banach metrics". We prove that these can always detect (virtual) homomorphisms in their metric-functional boundary. This provides a new way of searching for these kinds of decompositions, by creatively constructing suitable Banach metrics. All notions will be explained during the talk, no prior knowledge is required. Based on joint works with Liran Ron-George. (30 min. talk)

37. **Tianyi Zheng**
Title: Random walks on IMG and conformal dimension
Abstract: Conformal dimension was introduced in the late 1980s by P. Pansu; it is a natural invariant in the study of the geometry of hyperbolic spaces and their boundaries. In this talk we will discuss how conformal dimension can be used to study random walks on iterated monodromy groups, in particular, random walk entropy bounds when the limit set has Ahlfors-regular conformal dimension strictly less than 2. Joint work with N. Matte Bon and V. Nekrashevych. (30 min. talk)

38. **Fabio Zucca**
Title: A quick journey on stochastic orders and their applications to branching random walks
Abstract: The coupling technique is widely used in the theory of interacting particle systems. Our short journey starts from the classical stochastic order, and will take us to the germ order. We will see how the germ order can be successfully applied to branching random walks. This is a joint work with D. Bertacchi. (30 min. talk)