Calculation exercises for the exam ‘Fundamentals of Business Information Systems’

1) Travelling salesman problem (TSP)

Given the following symmetric travelling salesman problem answer the questions below:

		To city			
		A	B	C	D
From city	A	-	40	15	20
	B	40	-	35	25
	C	15	35	-	30
	D	20	25	30	-

Table 1.TSP- small example

1) How many solutions can be calculated?
2) Does the number of solutions change when the TSP becomes asymmetric?
3) What is the shortest path for the TSP given in the Table 1 above?
4) Assess the following statements:
    a. “If the travelling salesman always visits the nearest city he/she will always end up with the shortest path.”
    b. “If travelling salesman always visits the nearest city he/she will never end up with the shortest path.”
5) Consider a TSP with 28 cities – how would you obtain a good solution. Illustrate this on the small example above.

2) **Forecasting (SES and Holt)**

1) Given the demand (D) in Table 2, calculate the forecast for period (t) 8 with the following forecasting methods:
    - Moving Average (AVG) where the number of periods to include in the forecast (n) is 4.
    - Single Exponential Smoothing (SES) with smoothing parameter α=0.2
    - Holt’s Method with smoothing parameter α and β =0.2.

t	D	AVG (n=4)	SES (α=0.2)	Holt (α)	Holt (β)	Holt (α,β=0.2)
1	107					
2	130					
3	153					
4	149					
5	158					
6	160			149.60	13.32	
7	175		137.38			
8	?					

Table 2.Forecast example

2) Assume that the actual demand in period 8 is 172. Calculate the forecast quality of each approach with the following quality measures:
    - Mean absolute deviation (MAD)
    - Mean squared error (MSE)
    - Mean average percentage error (MAPE)
3) How do you decide on which forecast technique to use?
4) How do you decide on which quality measure to use?
5) Why should a company use a tracking signal to measure forecast quality?

3) Linear Programming

You invited some friends for a party. Therefore, you prepare two types of sandwiches:

Turkey sandwich: White bread, butter, 2 slices of cheese, 1 leaf of lettuce, 2 pieces of cucumber and 2 slices of turkey breast.
Speck sandwich: Dark rye bread, butter, 1 slice of cheese, 2 leaves of lettuce, 3 pieces of cucumber and 2 slices of bacon.

Your food stocks are as follows (Table 3):

Raw material	Amount	Unit
White/Dark rye Bread	more than enough	
Butter	more than enough	
Cheese	45	slices
Lettuce	48	leaves
Cucumber	60	slices
Turkey breast	24	pieces
Bacon	30	slices

Table 3. Food supply

Your friends are normally hearty eaters and not very picky – and you want them to be well supplied with food (make as much sandwiches as possible!). All grocery stores are already closed, but you still have enough time to make any kind of calculation:

1) Formulate an optimization model for the above stated problem.
2) Assume that you have calculated the optimal solution which is to make 12 'Turkey sandwiches' and 12 'Speck sandwiches'. Furthermore, you look at the dual prices of your food supplies and see the following dual prices:
    a. Dual price cheese = 0.
    b. Dual price lettuce = 0.
    c. Dual price cucumbers = 1/3.
    d. Dual price turkey breast = 1/6.
    e. Dual price bacon = 0.

    Interpret these results.
3) You obtain the following lower bounds (LB_dual) and upper bounds (UB_dual) of the dual prices:
    a. Cheese: LB_dual=36, UB_dual=infinite.
    b. Lettuce: LB_dual=36, UB_dual= infinite.
    c. Cucumbers: LB_dual=24, UB_dual=69.
    d. Turkey breast: LB_dual=15, UB_dual=37.5.
    e. Bacon: LB_dual=24, UB_dual= infinite.

    Interpret these results.
4) What are reduced cost of an objective value coefficient?
5) You obtain the following lower bounds (LB_obj) and upper bounds (UB_obj) for the objective value coefficients:
    a. Turkey sandwich: LB_obj=2/3, UB_obj= infinite.
    b. Bacon sandwich: LB_obj= 0, UB_obj=3/2.

    Interpret these results.
6) How would you alter your model if you:
    a. Want to make sure that there is no leftover bacon.
    b. Want to make sure that there is no leftover turkey breast.
    c. You know that 2/3 of your friends prefer 'Speck sandwiches'.

7) What would change if you want to guarantee that the solution is always integer?

Useful links:
TSP:
http://www.math.uwaterloo.ca/tsp/

Linear programming (in German):
http://www2.wiwi.uni-jena.de/Entscheidung/tenor/

Solutions (in short):

1) Travelling salesman problem (TSP)

1) 3 solutions.
2) Yes, the amount of solutions doubles (to 6 solutions).
3) The shortest path is A-D-B-C-A, since it is a symmetric TSP this is equal to the path A-C-B-D-A.
4)
   a. The statement describes a greedy heuristic (nearest neighbor) which yields quick results. The statement is false, since the heuristic does not always end up with the shortest path.
   b. This statement is also false, since there is a chance that heuristics yield the optimal solution. However, the likelihood decreases vastly as the problem size increases.
5) Describe at least two heuristics and show how they work given the example. E.g., nearest neighbor, k-opt.

2) Forecasting (SES and Holt)

1) AVG = 160.50; SES=144.90; Holt=179.14
2) AVG: MAD=11.50; MSE=132.25; MAPE=6.69%
   SES: MAD=27.10; MSE=734.19; MAPE=15.75%
   Holt: MAD=7.14; MSE=50.97; MAPE=4.15%
3) The selection depends on whether there is a seasonal pattern or a trend in the data (provide at least one eligible forecasting method for each scenario).
4) MAD shows absolute deviation and weighs all values equally; MSE is very sensitive to outliers; MAPE is easy to interpret, but might be misleading for time series with very small values.
5) Tracking signals are used to monitor your forecasts. They indicate if your forecasts are biased meaning that the calculated error lies beyond defined thresholds.

3) Linear Programming

1) x_1….number of Turkey sandwiches
   x_2….number of Speck sandwiches
   Objective function:

$$\max_{x_1 x_2} x_1 + x_2$$

   Subject to:

$$\begin{aligned}
&Cheese: && 2x_1 + 1x_2 \leq 45 \\
&Lettuce: && 1x_1 + 2x_2 \leq 48 \\
&Cucumber: && 2x_1 + 3x_2 \leq 60 \\
&Turkey: && 2x_1 \leq 24 \\
&Bacon: && 2x_2 \leq 30 \\
&NN: && x_1, x_2 \geq 0
\end{aligned}$$

2) If you make 24 sandwiches: 12 Turkey and 12 Speck sandwiches (optimal solution):
   a. Cheese is not a scarce raw material (you have some leftover), a dual price of zero means adding one unit does not increase your objective function.

b. See answer to a.
c. You do not have enough cucumber: The dual price of 1/3 tells you that by adding one slice of cucumber you could make an additional 1/3 of a (bacon) sandwich. (Why bacon? see answer to d.)
d. You do not have enough turkey breast: The dual price of 1/6 tells you that by adding one piece of turkey breast you could make 1/6 more sandwiches (in total).
   Explanation (not required at exam): with one more piece of turkey breast, you can make 12.5 Turkey and 11 2/3 Speck sandwiches, which is in total 24 1/6 sandwiches.
e. See answer to a.

3) The upper and lower bounds of the dual prices indicate the range within these dual prices are valid. Beyond these bounds, the optimal solution is defined by different constraints than in the original optimal solution:
   a. Lower bound: is the number of cheese slices that you need in the optimal solution. This means that if your supply of cheese drops below 36 slices, you would obtain a different optimal solution (cheese would become a scarce resource) and the dual prices change.
      Upper bound: all raw materials that have a dual price of zero have infinite upper bounds. This means that, since you already have leftovers in the optimal solution, adding more cheese does not increase the maximum number of sandwiches that you can make.
   b. See answer to a.
   c. Lower bound: If your supply of cucumbers drops below 24, you obtain a different optimal solution and different dual prices. Here, 'different optimal solution' means that although cucumber remains to be a scarce resource the other scarce resource in the optimal solution changes → turkey breast will not be a scarce resource any more.
      Upper bound: If your supply of cucumbers increases to more than 69, you obtain a different optimal solution and different dual prices. This means that beyond the upper bound, cucumbers are not a scarce resource any more.
   d. See answer to c.
   e. See answer to a.
4) Reduced costs indicate the value that the objective function coefficient must change before the value of the variable will be positive in the optimal solution. In our example the reduced costs of both (x_1 and x_2) variables are zero, since both variables are already positive in the optimal solution.
5) The upper and lower bounds of the objective function coefficients define the range in which the basis is unchanged:
   a. The lower bound of 2/3 means that, if the objective value coefficient of Turkey sandwiches drops to this value or below, the optimal solution is defined by different constraints. For example, if the objective function is altered to:
$$\max_{x_1 x_2} 0.5x_1 + x_2,$$
      then the optimal solution is not defined by the cucumber and turkey breast constraints any more (but the cucumber and bacon constraints).
      An infinite upper bound means that increasing the objective function coefficient of the Turkey sandwich does not change the definition of the optimal solution.

b. See answer to a

6)

a. Either increase the objective function coefficient of Speck sandwiches to a value of 3/2 (UB_obj) or simply add the constraint: $x_2 = 15$
b. You already have no leftover turkey breast, which is indicated by its non-zero dual price.
c. You should alter the objective function to resemble this relation, e.g.,

$$\max_{x_1 x_2} x_1 + 2x_2$$

7) The problem than becomes an integer linear program which is harder to solve.

For questions or errors with regard to the provided solutions, please contact stefan.haeussler@uibk.ac.at